

Leptonic CP Violation Induced by Approximately μ - τ Symmetric Seesaw Mechanism

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Assuming a minimal seesaw model with two heavy neutrinos (N), we examine effects of leptonic CP violation induced by approximate μ - τ symmetric interactions. As long as N is subject to the μ - τ symmetry, we can choose CP phases of Dirac mass terms without loss of generality in such a way that these phases arise from μ - τ symmetry breaking interactions. In the case that no phase is present in heavy neutrino mass terms, leptonic CP phases are controlled by two phases α and β . The similar consideration is extended to N blind to the μ - τ symmetry. It is argued that N subject (blind) to the μ - τ symmetry necessarily describes the normal (inverted) mass hierarchy. We restrict ourselves to μ - τ symmetric textures giving the tri-bimaximal mixing and calculate flavor neutrino masses to estimate CP-violating Dirac and Majorana phases as well as neutrino mixing angles as functions of α and β . Since α and β are generated by μ - τ symmetry breaking interactions, CP-violating Majorana phase tends to be suppressed and is found to be at most $\mathcal{O}(0.1)$ radian. On the other hand, CP-violating Dirac phase tends to show a proportionality to α or to β .

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I. INTRODUCTION

Recent extensive analysis on neutrino oscillations [1] has indicated almost maximal atmospheric neutrino mixing and large solar neutrino mixing as well as suppressed reactor neutrino mixing angle. These observed properties can well be understood by assuming a μ - τ symmetry in neutrino interactions [2]. Another interesting property of neutrinos, which has not yet been observed, is related to leptonic CP violation. The leptonic CP violation of the Dirac type is known to be absent in the μ - τ symmetric limit [3]. Therefore, to discuss physics of leptonic CP violation needs the μ - τ symmetry breaking in neutrino interactions.

Leptonic CP violation can be parameterized by one Dirac CP-violating phase (δ_{CP}) and three Majorana phases ($\phi_{1,2,3}$) [4] in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix $U_{PMNS} = U_\nu^0 K^0$ [5] with

$$U_\nu^0 = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

$$K^0 = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \quad (1)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ ($i, j=1,2,3$), as adopted by the Particle Data Group [6]. The Majorana CP-violating phases are determined by two combinations of $\phi_{1,2,3}$ such as $\phi_i - \phi_1$ ($i=1,2,3$). Since there are arbitrary phases of the flavor neutrinos, the phases of U_{PMNS} vary with these phases. The most general form of U_{PMNS} is given by U_ν and K [8] in place of U_ν^0 and K^0 :

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\rho} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}e^{-i\rho} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}e^{-i\rho} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix},$$

$$K = \text{diag}(e^{i\phi'_1}, e^{i\phi_2}, e^{i\phi_3}), \quad (2)$$

where $\delta_{CP} = \delta + \rho$ and $\phi_1 = \phi'_1 - \rho$, which will be used in this article.¹ Another aspect of the role of leptonic CP phases may lie in creation of the baryon asymmetry of the Universe [9] when the seesaw mechanism active at higher energies

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¹ The phases ρ and γ are redundant and can be removed by the redefinition of flavor neutrino masses. The resultant U_{PMNS} involves

[10] is responsible for generating neutrino masses [7, 11]. More precisely, the minimal seesaw mechanism based on two heavy neutrinos (N) provides the direct linkage between the high energy phases in the seesaw mechanism and the low energy phases defined by U_{PMNS} [12]. Therefore, we can predict the size of the low energy phases that yields the observed size of the baryon asymmetry of the Universe.

In the present article, we discuss μ - τ symmetry breaking effects in the minimal seesaw mechanism on flavor neutrino masses to study low energy CP violation [13]. The correlation between the low energy CP violation and its effect in leptogenesis will be discussed in a subsequent article. In Sec.II, our minimal seesaw model is described. In Sec.III, we calculate flavor neutrino masses in models based on N subject to the μ - τ symmetry where the normal mass hierarchy is realized and on N blind to the μ - τ symmetry where the inverted mass hierarchy is realized. How these mass hierarchies arise is discussed in the Appendix A. We restrict ourselves to mass terms of N without CP-violating phases. As a result, leptonic CP violation arises in general from the μ - τ symmetry breaking terms for the normal mass hierarchy. To compare our predictions with those for the inverted mass hierarchy, the same phase structure is assumed for the inverted mass hierarchy. To simplify our discussions, the μ - τ symmetric flavor neutrino mass texture is taken to describe the tri-bimaximal mixing [14], which predicts the consistent value of $\sin\theta_{12}$ with the observed data. We choose three textures to describe the normal and inverted mass hierarchies. The calculation of the flavor neutrino masses supplied by the seesaw mechanism is performed in the Appendix A. In Sec.IV, CP-violating phases as well as neutrino masses and mixing angles are calculated and results are shown in figures. A set of formula used in Sec.IV is summarized in the Appendix B. Final section is devoted to summary and discussions.

II. MODEL

Let us begin with defining superpotential for leptons (W) in the minimal seesaw model with three flavors of L (ℓ) as $SU(2)_L$ -doublets (singlets) and two flavors of N to be denoted by (N_μ, N_τ) as $SU(2)_L$ -singlets as well as two Higgses $H_{u,d}$:

$$W = e^{CT} \mathbf{Y}_\ell L H_d + N^{CT} \mathbf{Y}_\nu L H_u + \frac{1}{2} N^{CT} \mathbf{M}_R N^C \quad (3)$$

where \mathbf{Y}_ℓ and \mathbf{Y}_ν are Yukawa couplings and \mathbf{M}_R is a Majorana mass matrix of N . We can always choose the base, where \mathbf{Y}_ℓ is diagonal, which defines the charged leptons e , μ and τ . The coupling \mathbf{Y}_ν and the mass matrix \mathbf{M}_R are parameterized as follows:

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{\mu e} & h_{\mu\mu} & h_{\mu\tau} \\ h_{\tau e} & h_{\tau\mu} & h_{\tau\tau} \end{pmatrix}, \quad \mathbf{M}_R = \begin{pmatrix} \mathbf{M}_{R\mu\mu} & \mathbf{M}_{R\mu\tau} \\ \mathbf{M}_{R\mu\tau} & \mathbf{M}_{R\tau\tau} \end{pmatrix}. \quad (4)$$

Our μ - τ symmetry is defined by the invariance of W under the interchange $\nu_\mu \leftrightarrow -\sigma\nu_\tau$, where $\sigma = \pm 1$ will take care of the sign of $\sin\theta_{23}$.² This form of the interchange is based on the choice of $\sin\theta_{23} = \sigma/\sqrt{2}$ defined in U_{PMNS} of Eq.(1) in the μ - τ symmetric limit. It is readily seen that the corresponding mass matrix is $M^{(+)}$ of Eq.(A2), which has $(0, \sigma/\sqrt{2}, 1/\sqrt{2})^T$ as an eigenvector that in turn gives $\sin\theta_{23} = \sigma/\sqrt{2}$ from the third column of U_{PMNS} as long as this eigenvector is assigned to the third neutrino [16]. As a result, $M^{(+)}$ is invariant under $\nu_\mu \leftrightarrow -\sigma\nu_\tau$. It is convenient to introduce

$$\nu_\pm = \frac{\nu_\mu \pm (-\sigma\nu_\tau)}{\sqrt{2}}, \quad N_\pm^C = \frac{N_\mu^C \pm (-\sigma N_\tau^C)}{\sqrt{2}}, \quad (5)$$

to discuss property of neutrinos with respect to the μ - τ symmetry.

In addition to the assignment of N to be $N = (N_\mu, N_\tau)$, there are other cases, which contain N_e in models such as those based on $(N_e, N_{\mu,\tau})$. For N subject to the μ - τ symmetry, N can be either (N_+, N_-) or (N_e, N_-) while, for

$\delta + \rho$ and $\phi'_1 - \rho$, respectively, as Dirac and Majorana CP-violating phases. Although Eq.(2) contains 6 CP phases, strictly speaking, there are 7 CP phases in total [7]. In fact, there is an additional phase for the 2-3 rotation (τ) contributing to δ_{CP} as $\delta_{CP} = \delta + \rho + \tau$, which, however, can be removed by introducing a new definition: $\rho' = \rho + \tau/2$, $\gamma' = \gamma + \tau/2$ and $\delta' = \delta + \tau/2$. As a result, we end up with the same definition of δ_{CP} : $\delta_{CP} = \delta' + \rho'$. See Ref.[8] for more details. Therefore, the parameterization with δ' , ρ' , and γ' gives Eq.(2) as a general form of U_{PMNS} .

² This interchange should be replaced by $L_\mu \leftrightarrow -\sigma L_\tau$ and is consistently described if two extra Higgses $H'_{u,d}$ are introduced. Under the interchange, we require that $H'_{u,d} \rightarrow -H'_{u,d}$ while $H_{u,d} \rightarrow H_{u,d}$, where $\langle 0|H'_{u,d}|0 \rangle$ yields μ - τ symmetry breaking terms. The interplay of these Higgses supplies a μ - τ symmetric W and also accounts for the appearance of the badly broken μ - τ symmetry for the charged leptons and of the approximate μ - τ symmetry for neutrinos [15]. For the purpose of the present article, it is sufficient to use Eq.(3).

N blind to the μ - τ symmetry, N_{\pm} can be any two heavy neutrinos of N_e , N_{μ} and N_{τ} . To treat these cases, we use $N = (N_+, N_-)$ instead of $N = (N_{\mu}, N_{\tau})$. The coupling \mathbf{Y}_{ν} and the mass matrix \mathbf{M}_R are parameterized as follows:

$$\mathbf{Y}_{\nu} = \begin{pmatrix} h_{+e} & h_{+\mu} & h_{+\tau} \\ h_{-e} & h_{-\mu} & h_{-\tau} \end{pmatrix}, \quad \mathbf{M}_R = \begin{pmatrix} \mathbf{M}_{R++} & \mathbf{M}_{R+-} \\ \mathbf{M}_{R+-} & \mathbf{M}_{R--} \end{pmatrix}. \quad (6)$$

Another case with N subject to the μ - τ symmetry can be discussed by replacing (N_+, N_-) with (N_e, N_-) . We classify all cases by specifying couplings of the Yukawa interactions for neutrinos denoted by f 's, where neutrinos are expressed in terms of N_{\pm} and $\nu_{e,\pm}$. The corresponding lagrangian \mathcal{L}_{ν} is described by

$$\begin{aligned} -\mathcal{L}_{\nu} = & (f_{ee}\overline{N_e} + f_{+e}\overline{N_+} + f_{-e}\overline{N_-})\nu_e H_{u1} + (f_{e+}\overline{N_e} + f_{++}\overline{N_+} + f_{-+}\overline{N_-})\nu_+ H_{u1} \\ & + (f_{e-}\overline{N_e} + f_{+-}\overline{N_+} + f_{--}\overline{N_-})\nu_- H_{u1}. \end{aligned} \quad (7)$$

where H_{u1} is defined in $H_u = (H_{u1}, H_{u2})^T$. In each case, if N is subject to the μ - τ symmetry, N_+ can be $(N_{\mu} + (-\sigma)N_{\tau})/\sqrt{2}$ or N_e and N_- is just $(N_{\mu} - (-\sigma)N_{\tau})/\sqrt{2}$. On the other hand, if N is blind to the μ - τ symmetry, N_{\pm} can be any combinations of $N_{e,\mu,\tau}$. The difference of these assignments is absorbed into the definition of f 's, which are given by different Yukawa couplings in the starting superpotential expressed in terms of $N_{e,\mu,\tau}$ and $\nu_{e,\mu,\tau}$.³ It is sufficient to use the notation of N_{\pm} for the later discussions.

For a complex \mathbf{M}_R , phases of θ_{Rij} defined by $\mathbf{M}_{Rij} = \exp(i\theta_{Rij})\mathbf{M}_{ij}$ ($i, j = +, -$) can be transferred into the phase of \mathbf{M}_{R+-} , where \mathbf{M}_{ij} ($i, j = +, -$) are taken to be real for $-\pi/2 \leq \theta_{Rij} \leq \pi/2$. We here use $(+, -)$ as the suffix of \mathbf{M} , which should be replaced by other combinations such as $(e, -)$, appropriately. This phase becomes $\theta_{R+-} - (\theta_{R++} + \theta_{R--})/2 (\equiv \Theta_{+-})$ and \mathbf{M}_R is given by

$$\mathbf{M}_R = \begin{pmatrix} \mathbf{M}_{++} & e^{i\Theta_{+-}}\mathbf{M}_{+-} \\ e^{i\Theta_{+-}}\mathbf{M}_{+-} & \mathbf{M}_{--} \end{pmatrix}. \quad (8)$$

Without the loss of generality, we choose that $\mathbf{M}_{--} > \mathbf{M}_{++}$. The unitary matrix U that diagonalizes \mathbf{M}_R to give

$$\mathbf{M}_{\text{diag}} = U^* \mathbf{M}_R U^{\dagger} = \begin{pmatrix} M_1 e^{2i\varphi_1} & 0 \\ 0 & M_2 e^{2i\varphi_2} \end{pmatrix} \quad (9)$$

is

$$U = \begin{pmatrix} \cos \theta & -e^{i\omega} \sin \theta \\ e^{-i\omega} \sin \theta & \cos \theta \end{pmatrix}, \quad (10)$$

where

$$\omega = \arg(e^{i\Theta_{+-}}\mathbf{M}_{++} + e^{-i\Theta_{+-}}\mathbf{M}_{--}), \quad (11)$$

$$M_1 e^{2i\varphi_1} = \cos^2 \theta \mathbf{M}_{++} + e^{-2i\omega} \sin^2 \theta \mathbf{M}_{--} - 2 \cos \theta \sin \theta e^{i(\Theta_{+-} - \omega)} \mathbf{M}_{+-},$$

$$M_2 e^{2i\varphi_2} = \cos^2 \theta \mathbf{M}_{--} + e^{2i\omega} \sin^2 \theta \mathbf{M}_{++} + 2 \cos \theta \sin \theta e^{i(\Theta_{+-} + \omega)} \mathbf{M}_{+-}, \quad (12)$$

and

$$\tan \theta = \frac{2\mathbf{M}_{+-}r}{\mathbf{M}_{--} - \mathbf{M}_{++} + \sqrt{(\mathbf{M}_{--} - \mathbf{M}_{++})^2 + 4\mathbf{M}_{+-}^2 r^2}}, \quad (13)$$

The parameter r is given by

$$r = \frac{|e^{i\Theta_{+-}}\mathbf{M}_{++} + e^{-i\Theta_{+-}}\mathbf{M}_{--}|}{\mathbf{M}_{++} + \mathbf{M}_{--}}. \quad (14)$$

³ In a seesaw model with $N_{e,\mu,\tau}$, two light heavy neutrinos that effectively describes the minimal seesaw model discussed here are dynamically determined by mass terms of $N_{e,\mu,\tau}$ and can be any combination of $N_{e,\mu,\tau}$. However, if N in the minimal seesaw model is subject to the μ - τ symmetry, N should include N_- as a light heavy neutrino.

The phase ω is further expressed as

$$\tan \omega = \frac{\mathbf{M}_{++} - \mathbf{M}_{--}}{\mathbf{M}_{++} + \mathbf{M}_{--}} \tan \Theta_{+-}. \quad (15)$$

In the case of $N_- \rightarrow -N_-$ under the μ - τ symmetry transformation, we obtain that

$$\mathbf{M}_{+-} = 0, \quad (16)$$

leading to $\theta = 0$ in the μ - τ symmetric limit.

The coupling \mathbf{Y}_ν for (N_+, N_-) and $(\nu_e, \nu_\mu, \nu_\tau)$ can be parameterized to be:

$$\mathbf{Y}_\nu = \begin{pmatrix} h_{+e} & h_{+\mu} & h_{+\tau} \\ h_{-e} & h_{-\mu} & h_{-\tau} \end{pmatrix}, \quad (17)$$

and formally divided into two parts as $\mathbf{Y}_\nu = \mathbf{Y}_\nu^{(+)} + \mathbf{Y}_\nu^{(-)}$, which is just an identity, where the superscripts $(+)$ and $(-)$ of \mathbf{Y}_ν are, respectively, so chosen to stand for the μ - τ symmetry preserving and breaking terms. We obtain the following \mathbf{Y}_ν :

1. For N subject to the μ - τ symmetry,

$$\mathbf{Y}_\nu^{(+)} = \begin{pmatrix} h_{+e}^{(+)} & h_{+\mu}^{(+)} & -\sigma h_{+\mu}^{(+)} \\ 0 & h_{-\mu}^{(+)} & \sigma h_{-\mu}^{(+)} \end{pmatrix}, \quad \mathbf{Y}_\nu^{(-)} = \begin{pmatrix} 0 & h_{+\mu}^{(-)} & \sigma h_{+\mu}^{(-)} \\ h_{-e}^{(-)} & h_{-\mu}^{(-)} & -\sigma h_{-\mu}^{(-)} \end{pmatrix}, \quad (18)$$

where

$$\begin{aligned} h_{+e}^{(+)} &= f_{+e}, & h_{+\mu}^{(+)} &= \frac{f_{++}}{\sqrt{2}}, & h_{-\mu}^{(+)} &= \frac{f_{--}}{\sqrt{2}}, \\ h_{-e}^{(-)} &= f_{-e}, & h_{+\mu}^{(-)} &= \frac{f_{+-}}{\sqrt{2}}, & h_{-\mu}^{(-)} &= \frac{f_{-+}}{\sqrt{2}}, \end{aligned} \quad (19)$$

from

$$-\mathcal{L}_\nu = (f_{+e}\overline{N_+} + f_{-e}\overline{N_-})\nu_e H_{u1} + (f_{++}\overline{N_+} + f_{--}\overline{N_-})\nu_+ H_{u1} + (f_{+-}\overline{N_+} + f_{-+}\overline{N_-})\nu_- H_{u1}, \quad (20)$$

for $N = (N_+, N_-)$, and where

$$\begin{aligned} h_{+e}^{(+)} &= f_{ee}, & h_{+\mu}^{(+)} &= \frac{f_{e+}}{\sqrt{2}}, & h_{-\mu}^{(+)} &= \frac{f_{-+}}{\sqrt{2}}, \\ h_{-e}^{(-)} &= f_{-e}, & h_{+\mu}^{(-)} &= \frac{f_{e-}}{\sqrt{2}}, & h_{-\mu}^{(-)} &= \frac{f_{-e}}{\sqrt{2}}, \end{aligned} \quad (21)$$

from

$$-\mathcal{L}_\nu = (f_{ee}\overline{N_e} + f_{-e}\overline{N_-})\nu_e H_{u1} + (f_{e+}\overline{N_e} + f_{-+}\overline{N_-})\nu_+ H_{u1} + (f_{e-}\overline{N_e} + f_{-e}\overline{N_-})\nu_- H_{u1}, \quad (22)$$

for $N = (N_e, N_-)$. In the μ - τ symmetric limit, it is obvious to see that this case provides $\nu_e \nu_+$, $\nu_+ \nu_+$ and $\nu_- \nu_-$ as flavor neutrino mass terms and the phase of U is absent because of Eq.(16). Since the quantity $U^* \mathbf{Y}_\nu^{(+)} \mathbf{Y}_\nu^{(+)\dagger} U^T$ related to the leptogenesis turns out to be real, the leptogenesis requires μ - τ symmetry breaking couplings [17]. In the Yukawa interactions, we see that phases of (f_{+e}, f_{++}, f_{--}) in Eq.(20) or (f_{ee}, f_{e+}, f_{-+}) in Eq.(22) are, respectively, absorbed by adjusting phases of ν_e , ν_+ and ν_- . As a result, the phases arises solely from the μ - τ symmetry breaking couplings.

2. For N blind to the μ - τ symmetry,

$$\mathbf{Y}_\nu^{(+)} = \begin{pmatrix} h_{+e}^{(+)} & h_{+\mu}^{(+)} & -\sigma h_{+\mu}^{(+)} \\ h_{-e}^{(+)} & h_{-\mu}^{(+)} & -\sigma h_{-\mu}^{(+)} \end{pmatrix}, \quad \mathbf{Y}_\nu^{(-)} = \begin{pmatrix} 0 & h_{+\mu}^{(-)} & \sigma h_{+\mu}^{(-)} \\ 0 & h_{-\mu}^{(-)} & \sigma h_{-\mu}^{(-)} \end{pmatrix}, \quad (23)$$

where

$$\begin{aligned} h_{+e}^{(+)} &= f_{+e}, & h_{-e}^{(+)} &= f_{-e}, & h_{+\mu}^{(+)} &= \frac{f_{++}}{\sqrt{2}}, & h_{-\mu}^{(+)} &= \frac{f_{-+}}{\sqrt{2}}, \\ h_{+e}^{(-)} &= \frac{f_{+-}}{\sqrt{2}}, & h_{-\mu}^{(-)} &= \frac{f_{--}}{\sqrt{2}}, \end{aligned} \quad (24)$$

from the Yukawa interactions given by Eq.(20) for (N_+, N_-) and

$$\begin{aligned} h_{+e}^{(+)} &= f_{ee}, & h_{-e}^{(+)} &= f_{+e}, & h_{+\mu}^{(+)} &= \frac{f_{e+}}{\sqrt{2}}, & h_{-\mu}^{(+)} &= \frac{f_{++}}{\sqrt{2}}, \\ h_{+\mu}^{(-)} &= \frac{f_{e-}}{\sqrt{2}}, & h_{-\mu}^{(-)} &= \frac{f_{+-}}{\sqrt{2}}, \end{aligned} \quad (25)$$

from

$$-\mathcal{L}_\nu = (f_{ee}\overline{N}_e + f_{+e}\overline{N}_+) \nu_e H_{u1} + (f_{e+}\overline{N}_e + f_{++}\overline{N}_+) \nu_+ H_{u1} + (f_{e-}\overline{N}_e + f_{+-}\overline{N}_+) \nu_- H_{u1}, \quad (26)$$

for $N = (N_e, N_+)$. Similarly for the other cases. Since N can couple to ν_+ but not to ν_- in the μ - τ symmetric limit, flavor neutrino mass terms consist of $\nu_e \nu_+$ and $\nu_+ \nu_+$, which are phenomenologically favorable [18]. Furthermore, $U^* \mathbf{Y}_\nu^{(+)} \mathbf{Y}_\nu^{(+)\dagger} U^T$ becomes complex even in the μ - τ symmetric limit due to the presence of the μ - τ symmetric Majorana phase ω and may be preferable to the leptogenesis. In the Yukawa interactions, phases of the couplings of f 's can be absorbed into those associated with $\nu_{e,\pm}$. However, in general we cannot make the μ - τ symmetry preserving couplings real. We have to adjust the $\nu_{e,+}$ -couplings to be real by hand so that the μ - τ symmetry breaking associated with the ν_- -couplings supplies CP-phases.

The major conclusion is that N subject to the μ - τ symmetry has real μ - τ symmetry preserving couplings. Therefore, the leptonic CP-phases come from the μ - τ symmetry breaking couplings. For N blind to the μ - τ symmetry, the same situation arises only if we assume that phases are associated with the ν_- -couplings. In the μ - τ symmetric limit, the flavor neutrino masses can be parameterized to be:

$$\frac{1}{2} a_0 \nu_e \nu_e + b_0 \frac{\nu_e \nu_+ + \nu_+ \nu_e}{\sqrt{2}} + d_+ \nu_+ \nu_+ + d_- \nu_- \nu_-, \quad (27)$$

as in Eq.(A11) for N subject to the μ - τ symmetry, and

$$\frac{1}{2} a_0 \nu_e \nu_e + b_0 \frac{\nu_e \nu_+ + \nu_+ \nu_e}{\sqrt{2}} + d_+ \nu_+ \nu_+, \quad (28)$$

as in Eq.(A20) for N blind to the μ - τ symmetry, where a_0 , b_0 and d_\pm are mass parameters.

III. FLAVOR NEUTRINO MASS MATRIX

The seesaw mechanism generates the following Majorana neutrino mass matrix M_ν for flavor neutrinos:

$$M_\nu = -v^2 \mathbf{Y}_\nu^T U^T \mathbf{M}_{\text{diag}}^{-1} U \mathbf{Y}_\nu, \quad (29)$$

where $v = \langle 0 | H_{u1} | 0 \rangle$. We estimate effects of the leptonic CP violation provided by M_{ij} ($i, j = e, \mu, \tau$) defined in the Appendix A. For the sake of simplicity, we assume that there is no phase in \mathbf{M}_R .⁴

A. N subject to the μ - τ Symmetry

From Eq.(A8), we find that

$$\begin{aligned} M_{ee} &\approx -v^2 h_{+e}^{(+2)} M_1^{-1}, \\ M_{e\mu}^{(+)} &\approx -v^2 h_{+e}^{(+)} h_{+\mu}^{(+)} M_1^{-1}, \\ M_{e\mu}^{(-)} &\approx -v^2 \left[\left(h_{+\mu}^{(-)} - s h_{-\mu}^{(+)} \right) h_{+e}^{(+)} M_1^{-1} + \left(h_{-e}^{(-)} + s h_{+e}^{(+)} \right) h_{-\mu}^{(+)} M_2^{-1} \right], \\ M_{\mu\mu}^{(+)} &\approx -v^2 \left(h_{+\mu}^{(+2)} M_1^{-1} + h_{-\mu}^{(+2)} M_2^{-1} \right), \\ M_{\mu\mu}^{(-)} &\approx -2v^2 \left[\left(h_{+\mu}^{(-)} - s h_{-\mu}^{(+)} \right) h_{+\mu}^{(+)} M_1^{-1} + \left(h_{-\mu}^{(-)} + s h_{+\mu}^{(+)} \right) h_{-\mu}^{(+)} M_2^{-1} \right], \\ M_{\mu\tau} &\approx -v^2 (-\sigma) \left(h_{+\mu}^{(+2)} M_1^{-1} - h_{-\mu}^{(+2)} M_2^{-1} \right), \end{aligned} \quad (30)$$

⁴ We keep terms containing the phase in our calculations, whose results are shown in the Appendix and will be used in our future study.

up to the first order in the μ - τ symmetry breaking terms $h_{-e}^{(-)}$, $h_{+\mu}^{(-)}$, $h_{-\mu}^{(-)}$ and $s \propto \mathbf{M}_{+-}$. The results accord with our naive expectation that the μ - τ symmetric masses of M_{ee} , $M_{e\mu}^{(+)}$ and $M_{\mu\mu}^{(+)}$ are controlled by the μ - τ symmetric couplings. Since phases only arise from the μ - τ symmetry breaking couplings, we readily observe that this property shows the followings:

1. the μ - τ symmetric parts: M_{ee} , $M_{e\mu}^{(+)}$, $M_{\mu\mu}^{(+)}$ and $M_{\mu\tau}$ are real, and
2. the μ - τ symmetry breaking parts: $M_{e\mu}^{(-)}$ and $M_{\mu\mu}^{(-)}$ are complex,

within our approximation. It should be noted that M_{ee} , $M_{e\mu}^{(+)}$, $M_{\mu\mu}^{(+)}$ and $M_{\mu\tau}$ would have imaginary parts if second order terms of the μ - τ symmetry breaking are included. For example, in M_{ee} , its phase arises from $h_{-e}^{(-)}$ because $\omega = 0$. We then have a complex term proportional to $h_{+e}^{(+)}h_{-e}^{(-)}$, whose coefficient is cs which is the first order quantity because $s = 0$ in the μ - τ symmetric limit. Since $h_{e\mu}^{(-)}$ being itself is the first order quantity, the whole contribution from this term is the second order quantity, which can be safely neglected. In this way, we confirm that M_{ee} can be almost real. Similarly, we can confirm that $M^{(+)}$ itself is almost real. Therefore, main contributions to leptonic CP violation are given by $M_{e\mu}^{(-)}$ and $M_{\mu\mu}^{(-)}$. We will specify the phases of $M_{e\mu}^{(-)}$ and $M_{\mu\mu}^{(-)}$ by α and β , respectively.

As suggested by the above phase structure, the flavor neutrino mass matrix can be parameterized by M_ν : $M_\nu = M_\nu^{(+)} + M_\nu^{(-)}$ with

$$M_\nu^{(+)} = \begin{pmatrix} a_0 & b_0 & -\sigma b_0 \\ b_0 & d_0 & \sigma e_0 \\ -\sigma b_0 & \sigma e_0 & d_0 \end{pmatrix}, \quad M_\nu^{(-)} = \begin{pmatrix} 0 & b'_0 e^{i\alpha} & \sigma b'_0 e^{i\alpha} \\ b'_0 e^{i\alpha} & d'_0 e^{i\beta} & 0 \\ \sigma b'_0 e^{i\alpha} & 0 & -d'_0 e^{i\beta} \end{pmatrix}, \quad (31)$$

where a_0 , b_0 , d_0 , e_0 , b'_0 and d'_0 are all real and α and β are phases. We also use the notation: $a = a_0$, $b = b_0 + b'_0 e^{i\alpha}$, $c = -\sigma(b_0 - b'_0 e^{i\alpha})$, $d = d_0 + d'_0 e^{i\beta}$, $e = \sigma e_0$ and $f = d_0 - d'_0 e^{i\beta}$. In approximately μ - τ symmetric models, $b'_0 \approx 0$ and $d'_0 \approx 0$ are realized and yield the smallness of $\sin^2 \theta_{13}$ and the large mixing given by $\sin 2\theta_{23} \approx 1$ as a natural consequence. However, to understand the observed smallness of $\Delta m_\odot^2 / |\Delta m_{atm}^2|$, where $\Delta m_\odot^2 = m_2^2 - m_1^2 (> 0)$ and $\Delta m_{atm}^2 = m_3^2 - m_1^2$, needs an additional small parameter, which we call η .

The μ - τ symmetric mass matrix $M_\nu^{(+)}$ gives

$$\sin 2\theta_{12} = \frac{2\sqrt{2}b_0}{\sqrt{(d_0 - e_0 - a_0)^2 + 8b_0^2}}, \quad \cos 2\theta_{23} = \sin \theta_{13} = 0, \quad (32)$$

$$m_1 = \frac{a_0 + d_0 - e_0}{2} - \frac{\sqrt{2}b_0}{\sin 2\theta_{12}}, \quad m_2 = \frac{a_0 + d_0 - e_0}{2} + \frac{\sqrt{2}b_0}{\sin 2\theta_{12}}, \quad (33)$$

$$m_3 = d_0 + e_0,$$

as well as

$$\Delta m_\odot^2 = \frac{2\sqrt{2}(a_0 + d_0 - e_0)b_0}{\sin 2\theta_{12}}. \quad (34)$$

The spectrum contains one massless neutrino if the relation

$$a = \frac{b(bf - ce) - c(be - cd)}{df - e^2}, \quad (35)$$

for $df \neq e^2$, is satisfied. For $df = e^2$, we find that $be = cd$, leading to $m_3 = 0$ for $M_\nu^{(+)}$. Similarly,

$$e = \frac{bc \pm \sigma \sqrt{(b^2 - ad)(c^2 - af)}}{a}, \quad (36)$$

is another useful relation. For $M_\nu^{(+)}$, it is readily understood that

1. for the normal mass hierarchy, $m_1 = 0$ is obtained from Eq.(35) if $a_0 + d_0 - e_0 \geq 0$,
2. for the inverted mass hierarchy, $m_3 = 0$ is obtained from Eq.(36) giving $d_0 + e_0 = 0$.

The minimal seesaw mechanism in this case only allows the normal mass hierarchy [17] to account for the observed results as discussed in the Appendix A. One of the authors (M.Y.) has shown variety of textures, which are approximately μ - τ symmetric [18], from which we choose the following neutrino mass matrix:

$$M_\nu^{(+)} = m_0 \begin{pmatrix} p\eta & \eta & -\sigma\eta \\ \eta & 1 & \sigma(1-s\eta) \\ -\sigma\eta & \sigma(1-s\eta) & 1 \end{pmatrix} \quad (p = \frac{2}{s}), \quad (37)$$

giving

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{s-p}, \quad (38)$$

where η is to be estimated in Sec.IV to give $\eta(\sim \sqrt{\Delta m_\odot^2/|\Delta m_{atm}^2|}) = \mathcal{O}(10^{-1})$ and s (and p) are parameters of $\mathcal{O}(1)$. The condition of $\det(M_\nu) = 0$ is satisfied by

$$a = \frac{b(bf - ce) - c(be - cd)}{df - e^2} (= \frac{2\eta}{s} m_0 \text{ for } M_\nu^{(+)}). \quad (39)$$

The minimal seesaw model yields

$$\begin{aligned} m_0 &= -v^2 \left(h_{+\mu}^{(+2)} M_1^{-1} + h_{-\mu}^{(+2)} M_2^{-1} \right), \\ \eta &= \frac{h_{+e}^{(+)} h_{+\mu}^{(+)} M_1^{-1}}{h_{+\mu}^{(+2)} M_1^{-1} + h_{-\mu}^{(+2)} M_2^{-1}}, \\ s &= 2 \frac{h_{+\mu}^{(+)}}{h_{+e}^{(+)}}, \end{aligned} \quad (40)$$

where $p = 2/s$ is automatically satisfied as expected. Since $s = \mathcal{O}(1)$ and $\eta = \mathcal{O}(10^{-1})$, we have to adjust the parameters such that

$$\left| h_{+\mu}^{(+)} \right| \sim \left| h_{+e}^{(+)} \right| \ll \left| h_{-\mu}^{(+)} \right| \sqrt{|M_1/M_2|}, \quad (41)$$

equivalently,

$$\left| f_{+e}^{(+)} \right| \sim \left| f_{++}^{(+)} \right| \ll \left| f_{--}^{(+)} \right| \sqrt{|M_1/M_2|}, \quad (42)$$

which gives $\nu_- \nu_-$ as a dominant mass term.

B. N blind to the μ - τ Symmetry

From Eq.(A18), we find that

$$\begin{aligned} M_{ee} &= -v^2 \left[\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right)^2 M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 M_2^{-1} \right], \\ M_{e\mu}^{(+)} &= -v^2 \left[\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right) M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(sh_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) M_2^{-1} \right], \\ M_{e\mu}^{(-)} &= -v^2 \left[\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right) \left(ch_{+\mu}^{(-)} - sh_{-\mu}^{(-)} \right) M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(sh_{+\mu}^{(-)} + ch_{-\mu}^{(-)} \right) M_2^{-1} \right], \\ M_{\mu\mu}^{(+)} &\approx -v^2 \left[\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1} \right], \\ M_{\mu\mu}^{(-)} &= -2v^2 \left[\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right) \left(ch_{+\mu}^{(-)} - sh_{-\mu}^{(-)} \right) M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right) \left(ch_{-\mu}^{(-)} + sh_{+\mu}^{(-)} \right) M_2^{-1} \right], \\ M_{\mu\tau} &\approx -v^2 (-\sigma) \left[\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1} \right], \end{aligned} \quad (43)$$

up to the first order in the μ - τ symmetry breaking terms $h_{+\mu}^{(-)}$, and $h_{-\mu}^{(-)}$.

The minimal seesaw mechanism forbids the normal mass hierarchy to account for the observed results as discussed in the Appendix A. There are two types of neutrino mass textures [18].

1. As the inverted mass hierarchy I (with $m_1 \sim m_2$),

$$M_\nu^{(+)} = m_0 \begin{pmatrix} 2 - p\eta & \eta & -\sigma\eta \\ \eta & 1 & -\sigma \\ -\sigma\eta & -\sigma & 1 \end{pmatrix}, \quad (44)$$

$$(45)$$

leading to

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{p}. \quad (46)$$

The condition of $\det(M_\nu) = 0$ is satisfied by

$$e = \frac{bc - \sigma\sqrt{(b^2 - ad)(c^2 - af)}}{a} (= -\sigma d_0 \text{ for } M_\nu^{(+)}). \quad (47)$$

These parameters are related to those in the seesaw mechanism given by

$$\begin{aligned} m_0 &= -v^2 \left[\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1} \right], \\ \eta &= \frac{\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right) M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(sh_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) M_2^{-1}}{\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1}}, \\ p &= \frac{\left[2 \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 - \left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right)^2 \right] M_1^{-1} + \left[2 \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 - \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 \right] M_2^{-1}}{\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right) M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(sh_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) M_2^{-1}}. \end{aligned} \quad (48)$$

2. As the inverted mass hierarchy II (with $m_1 \sim -m_2$),

$$M_\nu^{(+)} = m_0 \begin{pmatrix} -(2 - \eta) & q & -\sigma q \\ q & 1 & -\sigma \\ -\sigma q & -\sigma & 1 \end{pmatrix}, \quad (49)$$

leading to

$$\tan 2\theta_{12} = \frac{2\sqrt{2}q}{4 - \eta}. \quad (50)$$

The condition of $\det(M_\nu) = 0$ is satisfied by

$$e = \frac{bc + \sigma\sqrt{(b^2 - ad)(c^2 - af)}}{a} (= -\sigma d_0 \text{ for } M_\nu^{(+)}). \quad (51)$$

These parameters are related to those in the seesaw mechanism given by

$$\begin{aligned} m_0 &= -v^2 \left[\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1} \right], \\ \eta &= \frac{\left[2 \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 + \left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right)^2 \right] M_1^{-1} + \left[2 \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 \right] M_2^{-1}}{\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1}}, \\ q &= \frac{\left(ch_{+e}^{(+)} - sh_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right) M_1^{-1} + \left(sh_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(sh_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) M_2^{-1}}{\left(ch_{+\mu}^{(+)} - sh_{-\mu}^{(+)} \right)^2 M_1^{-1} + \left(ch_{-\mu}^{(+)} + sh_{+\mu}^{(+)} \right)^2 M_2^{-1}}, \end{aligned} \quad (52)$$

The parameter η is to be estimated in Sec.IV to give $\eta(\sim \Delta m_\odot^2/|\Delta m_{atm}^2|) = \mathcal{O}(10^{-2})$ and p and q are parameters of $\mathcal{O}(1)$. We have to adjust sizes of the parameters of the seesaw to account for the neutrino mass spectrum.

IV. CP PHASES

In this section, we discuss how leptonic CP phases are generated by M_ν of Eq.(31). For N blind to the μ - τ symmetry, the phase structure of Eq.(31) is not a general consequence. So, we choose a specific parameter set so that phases only arise from $M_{e\mu}^{(-)}$ and $M_{\mu\mu}^{(-)}$. In other words, phases should be associated with the couplings of ν_- . Furthermore, there have been arguments that the renormalization effects are significant for the inverted mass hierarchy [19], which is the case of N blind to the μ - τ symmetry. However, the smallness of $\sin^2 \theta_{13}$ is not disturbed because it is a result of the approximate μ - τ symmetry but the CP-violating phases may receive significant distortion. This subject will be discussed elsewhere. For a moment, we show the case of the inverted mass hierarchy to make a comparison with the case of the normal mass hierarchy.

Our seesaw model has four phases from three Yukawa couplings and one Majorana phase of heavy neutrinos corresponding to one Dirac phase and three Majorana phase where one overall Majorana phase is redundant. Therefore, three CP-violating phases are present. This number is consistent with the general result of the seesaw model with N -flavor and M -heavy neutrinos, giving $N(M-1)$. Since the μ - τ symmetry breaking is so small that terms up to its first order contributions as in Eq.(30) can well describe neutrino phenomenology, two phases α and β become active and other phases associated with second-order contributions are safely neglected. The CP-phases including $\delta_{CP} = \delta + \rho$ are in general complicated functions of α and β . These two phases are the sources of the Dirac and Majorana phases in U_{PMNS} . However, we will see that when mass hierarchies are taken into account, ρ is found to be small and the dependence of α and β can be derived to give $\delta_{CP} \sim \alpha$ for the normal mass hierarchy and $\delta_{CP} \sim -\alpha$ the inverted mass hierarchy (with $m_1 \sim m_2$). These features can be viewed in the figures of δ_{CP} to be presented.

A. Estimations

The Dirac CP-violating phase is given by $\delta + \rho$ evaluated from Eq.(B3) in the Appendix B, from which we obtain that

$$c_{13}X \approx \sqrt{2} \left(b_0 (a_0 + d_0 - e_0) + b'_0 d'_0 e^{i(\beta-\alpha)} + (\Delta + i\gamma) (a_0 b'_0 e^{i\alpha} + b'_0 e^{-i\alpha} (d_0 + e_0) + b_0 d'_0 e^{i\beta}) \right), \quad (53)$$

$$Y \approx \sqrt{2}\sigma \left(-(\Delta - i\gamma) \left(b_0 (a_0 + d_0 - e_0) + b'_0 d'_0 e^{i(\beta-\alpha)} \right) + a_0 b'_0 e^{i\alpha} + b'_0 e^{-i\alpha} (d_0 + e_0) + b_0 d'_0 e^{i\beta} \right), \quad (54)$$

where the approximation is due to $|\gamma| \ll 1$, $\cos \theta_{23} = (1 + \Delta)/\sqrt{2}$ and $\sin \theta_{23} = \sigma(1 - \Delta)/\sqrt{2}$ for $|\Delta| \ll 1$. The phases δ and ρ are calculated from

$$\delta = -\arg(Y), \quad \rho = \arg(X). \quad (55)$$

From Eq.(53), it is expected that $\rho \approx 0$ if $b_0 (a_0 + d_0 - e_0)$ is not suppressed. This expectation is valid in the two textures of the inverted mass hierarchy; however, ρ may not be suppressed in the normal mass hierarchy because $a_0 + d_0 - e_0 \approx 0$ by Eq.(37). The parameters γ and Δ are estimate to be:

$$\gamma \approx \frac{4 (b_0 b'_0 \sin \alpha - e_0 d'_0 \sin \beta) - \sigma \sin \theta_{13} \sin 2\theta_{12} \sin (\rho + \delta) \Delta m_\odot^2}{2\Delta m_{atm}^2}, \quad (56)$$

$$\Delta \approx -\frac{4 (b_0 b'_0 \cos \alpha + d_0 d'_0 \cos \beta) + \sigma \sin \theta_{13} \sin 2\theta_{12} \cos (\rho + \delta) \Delta m_\odot^2}{2\Delta m_{atm}^2}. \quad (57)$$

The CP-violating phase $\delta + \rho$ can be numerically obtained from Eqs.(53) and (54) by using iteration, where $\Delta \pm i\gamma$ is given by $\gamma \approx 2 (b_0 b'_0 \sin \alpha - e_0 d'_0 \sin \beta) / \Delta m_{atm}^2$ and $\Delta \approx -2 (b_0 b'_0 \cos \alpha + d_0 d'_0 \cos \beta) / \Delta m_{atm}^2$ as a first trial.

The CP-violating Majorana phase is estimated from Eq.(B11) for $m_{1,2,3}$. We have assured, as expected, that $m_1 = 0$ for the normal mass hierarchy and $m_3 = 0$ for the inverted mass hierarchy within our numerical accuracy. From Eq.(B11), we find that

$$\begin{aligned} m_2 e^{-2i\phi_2} &\approx \frac{2\sqrt{2}}{\sin 2\theta_{12}} \left[(1 + i\gamma\Delta) b_0 + (\Delta + i\gamma) b'_0 e^{i\alpha} \right] e^{i\rho}, \\ m_3 e^{-2i\phi_3} &\approx \lambda_3 + s_{13}^2 (\lambda_3 - e^{2i\delta} a), \end{aligned} \quad (58)$$

where $\lambda_3 \approx d_0 + e_0 - 2i(2\Delta\gamma d_0 - (\gamma + i\Delta)e^{i\beta}d'_0)$, for the normal mass hierarchy with $m_1 = 0$, and

$$\begin{aligned} m_1 e^{-2i\phi_1} &\approx \frac{ae^{2i\rho} + d_0 - e_0 + 2(2i\Delta\gamma d_0 + (\Delta + i\gamma)e^{i\beta}d'_0)}{2} \\ &\quad - \frac{\sqrt{2}}{\sin 2\theta_{12}} [(1 + i\gamma\Delta)b_0 + (\Delta + i\gamma)b'_0 e^{i\alpha}] e^{i\rho}, \\ m_2 e^{-2i\phi_2} &\approx \frac{ae^{2i\rho} + d_0 - e_0 + 2(2i\Delta\gamma d_0 + (\Delta + i\gamma)e^{i\beta}d'_0)}{2} \\ &\quad + \frac{\sqrt{2}}{\sin 2\theta_{12}} [(1 + i\gamma\Delta)b_0 + (\Delta + i\gamma)b'_0 e^{i\alpha}] e^{i\rho}, \end{aligned} \quad (59)$$

for the inverted mass hierarchy with $m_3 = 0$. It should be noted that the size of $\phi_{1,2}$ is generically small since the nonvanishing $m_{1,2}$ for the inverted mass hierarchy start with the unsuppressed μ - τ symmetric terms.

To perform our numerical calculations, we use exact formula without approximation: Eq.(B3) for $\theta_{12,13}$, δ and ρ , Eq.(B8) for θ_{23} Eq.(B10) for γ and Eq.(B11) for $\phi_{1,2,3}$. The tri-bimaximal neutrino mixing [14] is assumed for $M_\nu^{(+)}$ and is realized by

1. $s = 2$ in Eq.(37) for the normal mass hierarchy,
2. $p = 1$ in Eq.(44) for the inverted mass hierarchy I (with $m_1 \sim m_2$),
3. $q = 4 - \eta$ in Eq.(49) for the inverted mass hierarchy II (with $m_1 \sim -m_2$).

We estimate the CP-violating phases $\delta + \rho$ and $\phi_1 - \phi_2$ (or $\phi_2 - \phi_3$) as well as the mixing angles as functions of α and β for given values of $|\Delta m_{atm}^2| = 2.59 \sim 2.61 (\times 10^{-3} \text{ eV}^2)$ and $\Delta m_\odot^2 = 7.87 \sim 7.93 (\times 10^{-5} \text{ eV}^2)$, which are taken to sit on values around their center values in the recent data:

$$|\Delta m_{atm}^2| = (2.6 \pm 0.2) \times 10^{-3} \text{ eV}^2, \quad \Delta m_\odot^2 = (7.9 \pm 0.3) \times 10^{-5} \text{ eV}^2, \quad (60)$$

as the allowed 1σ ranges [1]. Our iteration starts with the calculation of m_0 by using D_+ in Eq.(B6) for given values of Δm_{atm}^2 and Δm_\odot^2 and these given values are compared with their computed values from our formula for a consistency check.

B. Predictions

Before performing numerical calculations, we show our predictions from three textures:

1. for the normal mass hierarchy ($p = 2/s$ with $s = 2$),

$$\frac{\Delta m_\odot^2}{|\Delta m_{atm}^2|} \approx \frac{(s+p)\eta^2}{\sqrt{2}\sin 2\theta_{12}}, \quad (61)$$

suggesting that $\eta = \mathcal{O}(10^{-1})$, and

$$c_{13}X \approx \sqrt{2} \left(m_0^2(s+p)\eta^2 + b'_0 d'_0 e^{i(\beta-\alpha)} + 2m_0(\Delta + i\gamma)b'_0 e^{-i\alpha} \right), \quad Y \approx \sqrt{2}\sigma m_0 b'_0 e^{-i\alpha}, \quad (62)$$

leading to

$$\rho = \text{arbitrary}, \quad \delta \approx \alpha, \quad (63)$$

where we will numerically find that the term proportional to η^2 gives dominated contribution in $c_{13}X$, which result in $\rho \approx 0$, and

$$\begin{aligned} \sin \theta_{13} e^{-i\delta} &\approx \frac{Y}{\Delta m_{atm}^2}, \\ \gamma &\approx 2(\eta b'_0 \sin \alpha - d'_0 \sin \beta) \frac{m_0}{\Delta m_{atm}^2} - \frac{1}{2} \sigma s_{13} \sin(\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_\odot^2}{\Delta m_{atm}^2}, \\ \cos 2\theta_{23} (\approx 2\Delta) &\approx - \left(4(\eta b'_0 \cos \alpha + d'_0 \cos \beta) \frac{m_0}{\Delta m_{atm}^2} + \sigma s_{13} \cos(\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \right), \end{aligned} \quad (64)$$

as well as

$$m_2 e^{-2i\phi_2} \approx \frac{2\sqrt{2}\eta m_0 e^{i\rho}}{\sin 2\theta_{12}}, \quad m_3 e^{-2i\phi_3} \approx (2 - s\eta) m_0 - 2(2i\Delta\gamma m_0 + (\Delta - i\gamma) e^{i\beta} d'_0), \quad (65)$$

leading to $|\Delta m_{atm}^2| \approx 4m_0^2$ and

$$\phi \approx -\frac{\rho}{4}, \quad (66)$$

for $\rho \approx 0$,

2. for the inverted mass hierarchy I (with $m_1 \sim m_2$ with $p = 1$),

$$\frac{\Delta m_{\odot}^2}{|\Delta m_{atm}^2|} \approx \frac{2\sqrt{2}\eta}{\sin 2\theta_{12}}, \quad (67)$$

suggesting that $\eta = \mathcal{O}(10^{-2})$, and

$$c_{13}X \approx 4\sqrt{2}m_0^2\eta, \quad Y \approx 2\sqrt{2}\sigma m_0 b'_0 e^{i\alpha}, \quad (68)$$

leading to

$$\rho \approx 0, \quad \delta \approx -\alpha, \quad (69)$$

and

$$\begin{aligned} \sin \theta_{13} e^{-i\delta} &\approx \frac{Y}{\Delta m_{atm}^2}, \\ \gamma &\approx 2(\eta b'_0 \sin \alpha + d'_0 \sin \beta) \frac{m_0}{\Delta m_{atm}^2} - \frac{1}{2} \sigma s_{13} \sin(\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2}, \\ \cos 2\theta_{23} (\approx 2\Delta) &\approx - \left(4(\eta b'_0 \cos \alpha + d'_0 \cos \beta) \frac{m_0}{\Delta m_{atm}^2} + \sigma s_{13} \cos(\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \right), \end{aligned} \quad (70)$$

as well as

$$\begin{aligned} m_1 e^{-2i\phi_1} &\approx \left(1 + e^{2i\rho} - \frac{p\eta e^{2i\rho}}{2} - \frac{\sqrt{2}\eta e^{i\rho}}{\sin 2\theta_{12}} \right) m_0, \\ m_2 e^{-2i\phi_2} &\approx \left(1 + e^{2i\rho} - \frac{p\eta e^{2i\rho}}{2} + \frac{\sqrt{2}\eta e^{i\rho}}{\sin 2\theta_{12}} \right) m_0, \end{aligned} \quad (71)$$

leading to $|\Delta m_{atm}^2| \approx m_0^2$ and

$$\phi = 0, \quad (72)$$

up to $\mathcal{O}(\rho^2)$ and

3. for the inverted mass hierarchy II (with $m_1 \sim -m_2$ with $q = 4 - \eta$),

$$\frac{\Delta m_{\odot}^2}{|\Delta m_{atm}^2|} \approx \frac{\sqrt{2}\eta \sin 2\theta_{12}}{q}, \quad (73)$$

suggesting that $\eta = \mathcal{O}(10^{-2})$, and

$$c_{13}X \approx 2\sqrt{2}m_0^2\eta q, \quad Y \approx -\sqrt{2}\sigma m_0 (2b'_0 e^{i\alpha} - qd'_0 e^{i\beta}), \quad (74)$$

leading to

$$\rho \approx 0, \quad \delta \approx \text{arbitrary}, \quad (75)$$

and

$$\begin{aligned}
\sin \theta_{13} e^{-i\delta} &\approx \frac{Y}{\Delta m_{atm}^2}, \\
\gamma &\approx 2 (qb'_0 \sin \alpha + d'_0 \sin \beta) \frac{m_0}{\Delta m_{atm}^2} - \frac{1}{2} \sigma s_{13} \sin (\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2}, \\
\cos 2\theta_{23} (\approx 2\Delta) &\approx -4 (qb'_0 \cos \alpha + d'_0 \cos \beta) \frac{m_0}{\Delta m_{atm}^2} + \sigma s_{13} \cos (\rho + \delta) \sin 2\theta_{12} \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2},
\end{aligned} \tag{76}$$

as well as

$$\begin{aligned}
m_1 e^{-2i\phi_1} &\approx \left(\frac{2(1 - e^{2i\rho}) + \eta e^{2i\rho}}{2} - \frac{\sqrt{2}q e^{i\rho}}{\sin 2\theta_{12}} \right) m_0, \\
m_2 e^{-2i\phi_2} &\approx \left(\frac{2(1 - e^{2i\rho}) + \eta e^{2i\rho}}{2} + \frac{\sqrt{2}q e^{i\rho}}{\sin 2\theta_{12}} \right) m_0,
\end{aligned} \tag{77}$$

leading to $|\Delta m_{atm}^2| \approx m_0^2$ and

$$\phi \approx -\frac{\sin 2\theta_{12}}{\sqrt{2}q} \rho, \tag{78}$$

which becomes $-\rho/6$ for $\sin 2\theta_{12} \approx 2\sqrt{2}/3$ and $q \approx 4$.

It is expected that $\sin \theta_{13}$ has no distinct dependence of α and β because $\sin \theta_{13}$ is determined by the radial part of Y whose phase from α and β controls δ .

The predictions are depicted in FIG.1-FIG.14 for the normal and inverted mass hierarchies. The effect of the sign of σ is irrelevant because it always accompanies $\sin \theta_{13}$. The gross features of the figures for the Dirac CP-violating phase δ_{CP} accord with our results Eqs.(63), (69) and (75). Namely,

1. for the normal mass hierarchy, the crude proportionality of δ_{CP} to α shown in FIG.1 is accounted by Eq.(63) with $\rho \sim 0$ and the effect of ρ gives scattered plots around the line $\delta_{CP} \propto \alpha$;
2. for the inverted mass hierarchy I (with $m_1 \sim m_2$), the clear proportionality of δ_{CP} to α is shown in FIG.5 as suggested by Eq.(69);
3. for the inverted mass hierarchy II (with $m_1 \sim -m_2$), the proportionality of δ_{CP} to β can be seen as sharp edges in FIG.9 and is suggested by Eq.(69) for the region of $b'_0 \sim 0$;
4. In FIG.13, the Dirac CP-violating phase is found to be proportional to δ . This behavior indicates that $\rho \sim 0$. This is because X in Eq.(B3) starts with the μ - τ symmetric contribution, which can be taken to be real, and, then, the phase ρ starts with the μ - τ breaking contribution, which generically suppressed, giving $\rho \sim 0$.

The CP-violating Majorana phases ϕ are predicted

1. in FIG.2, FIG.6 and FIG.10, where the CP-violating Majorana phase almost vanishes for the inverted mass hierarchy I as predicted in Eq.(71);
2. in FIG.14, where the CP-violating Majorana phase is found to be proportional to ρ . This feature can be roughly understood because of $\rho \sim 0$ in Eq.(B11) and the contribution of δ in the difference of Majorana phases almost vanish. Namely, we can estimate that $\phi \propto \rho$. More precisely, our predictions Eqs.(66), (72) and (78) on ϕ are consistent with the behavior of these figures.

The mixing angle θ_{13} satisfies the constraints:

1. $\sin \theta_{13} \lesssim 0.05$ for the normal mass hierarchy;
2. $\sin \theta_{13} \lesssim 0.09$ for the inverted mass hierarchy I (with $m_1 \sim m_2$);
3. $\sin \theta_{13} \lesssim 0.05$ and $\sin \theta_{13} \sim 0.05$ around $\alpha \sim \beta \sim \pi$ for the inverted mass hierarchies II (with $m_1 \sim -m_2$),

as can be seen from FIG.3, FIG.7 and FIG.11 and $\tan^2 \theta_{23} > 1$

1. if $0 \leq \beta \leq \pi/2$ for the normal mass hierarchy;
2. if $\pi/2 \leq \beta \leq \pi$ for the inverted mass hierarchy I (with $m_1 \sim m_2$);
3. $\pi/2 \leq \alpha \leq \pi$ for the inverted mass hierarchies II (with $m_1 \sim -m_2$),

as in FIG.4, FIG.8 and FIG.12.

V. SUMMARY

We have estimated CP-violating phases as well as mixing angles in the approximately $\mu - \tau$ symmetric minimal seesaw model. When heavy neutrino mass terms are real, we have shown that CP-violating phases are determined by $\mu - \tau$ symmetry breaking phases in the neutrino Yukawa couplings as long as heavy neutrinos are transformed under the discrete $\mu - \tau$ symmetry group. As a result, phases in the flavor neutrino masses are expressed in terms of two phases α and β as given by Eq.(31). On the other hand, such a property is not a general one if heavy neutrinos are not transformed. We have assumed the same phases α and β to compare our predictions. Furthermore, we have found that the normal mass hierarchy is permitted if heavy neutrinos are subject to the $\mu - \tau$ symmetry giving a constraint of $M_{e\mu}^{(+2)} \approx M_{ee}M_{\mu\mu}$, which is used to exclude the inverted mass hierarchy and that the inverted mass hierarchy is permitted if the heavy neutrinos are blind to the $\mu - \tau$ symmetry giving a constraint of $M_{\mu\tau} \approx -\sigma M_{\mu\mu}^{(+)}$, which is used to exclude the normal mass hierarchy. The restriction on the mass hierarchy is a general consequence of approximately $\mu - \tau$ symmetric minimal seesaw models as long as no phases are present in heavy neutrinos.

We have also presented three textures, which give the consistent results with the current neutrino oscillation data: one describes the normal mass hierarchy as determined by Eq.(37) and the other two describe the inverted mass hierarchy as determined by Eq.(44) and Eq.(49). Each textures have a small parameter η to explain the smallness of the ratio of mass squared differences $\Delta m_{\odot}/\Delta m_{atm}(\equiv R)$, which is $\mathcal{O}(\sqrt{R})$ ($\mathcal{O}(R)$) for the normal (inverted) mass hierarchy. The Dirac CP-violating phase is predicted from our formula Eq.(55) to yield $\delta_{CP} = \rho + \delta$. Because of $\rho \sim 0$, we have found that the phase δ_{CP} is determined by α as $\delta_{CP} \approx \alpha$ as in Eq.(63) for the normal mass hierarchy and $\delta_{CP} \approx -\alpha$ for the inverted mass hierarchy I (with $m_1 \sim m_2$) as in Eq.(69) while δ_{CP} shows no dependence of α but a certain dependence of β for the inverted mass hierarchy II (with $m_1 \sim -m_2$) as in Eq.(75). The numerical calculation is performed to make definite predictions, whose results are shown in FIG.1-FIG.14. We have observed that

1. The Dirac CP-phase δ_{CP} turns out to have a crude proportionality to α in the normal mass hierarchy as FIG.1, a clear proportionality to α in the inverted mass hierarchy I (with $m_1 \sim m_2$) as FIG.5 and an proportionality to β (for $b'_0 \sim 0$) for inverted mass hierarchy II (with $m_1 \sim -m_2$) as FIG.9.
2. The Majorana CP-violating phase ϕ is found to be suppressed since its main contributions arise from $\mu - \tau$ symmetry breaking terms and is estimated to be: $\phi \approx -\rho/4$ for the normal mass hierarchy, $\phi \approx 0$ for the inverted mass hierarchy I (with $m_1 \sim m_2$) and $\phi \approx -\rho/6$ for the inverted mass hierarchy II (with $m_1 \sim -m_2$) as in FIG.14,
3. Our phases δ and ρ , respectively, yield main contributions to δ_{CP} and ϕ with $\rho \sim 0$ as in FIG.13 and 14, whose behaviors accord with our theoretical expectation.

From these observations, we expect that the size of the CP-violating Majorana phase can be enhanced if we include the phase of the heavy neutrinos Θ_{+-} as in Eq.(8). For the inverted mass hierarchy, we may relax our assumption that the $\mu - \tau$ symmetric terms are set to be real.

Last but not least, we have to comment on the effective neutrino mass $m_{\beta\beta}$ [20] used in the detection of the absolute neutrino mass [21]. In our textures, $m_{\beta\beta}$ corresponds to the flavor mass of $e^{2i\rho}M_{ee}$ as in Eq.(B16). As stated in the Appendix, it is not M_{ee} defined in Eq.(A1) that can be compared with experimental parameters, which are based on Eq.(1). In our case, since Eq.(2) is an appropriate matrix, which should be transformed into Eq.(1). In the course of this transformation, M_{ee} in Eq.(A1) is changed to $e^{2i\rho}M_{ee}$, which is parameterized to be $e^{2i\rho}a$ for a real a . Therefore, in principle the phase ρ has a chance to be measured. It is known that $|m_{\beta\beta}|$ is suppressed for the normal mass hierarchy, where the suppression factor η appears in our texture, and is estimated to be $a(\sim \eta m_0) \sim \sqrt{\Delta m_{\odot}^2}$ with $|\Delta m_{atm}^2| \approx 4m_0^2$ while, for the inverted mass hierarchy, $|m_{\beta\beta}| \approx 2m_0$ with $|\Delta m_{atm}^2| \approx m_0^2$ are obtained.

The predicted behaviors of CP phases are those at the seesaw scale. Radiative corrections to CP-phases should be evaluated to yield their observed values at the low-energy scale. Since these corrections are expected to be significant for the inverted mass hierarchy, we will estimate these corrections in the future publication. Furthermore since we know CP phases of the Yukawa couplings of neutrinos that can be inferred from the predicted Dirac and Majorana CP-violating phases, we can discuss how the leptogenesis is realized without referring to a specific form of flavor neutrino mass matrix but only with referring to more general framework of the $\mu - \tau$ symmetry breaking.

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APPENDIX A: FLAVOR NEUTRINO MASSES FROM SEESAW MECHANISM

In this Appendix, we evaluate flavor neutrino masses M_{ij} ($i, j = e, \mu, \tau$) that form a mass matrix M_ν :

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = M^{(+)} + M^{(-)}. \quad (\text{A1})$$

with

$$M^{(+)} = \begin{pmatrix} M_{ee} & M_{e\mu}^{(+)} & -\sigma M_{e\mu}^{(+)} \\ M_{e\mu}^{(+)} & M_{\mu\mu}^{(+)} & M_{\mu\tau} \\ -\sigma M_{e\mu}^{(+)} & M_{\mu\tau} & M_{\mu\mu}^{(+)} \end{pmatrix}, \quad M^{(-)} = \begin{pmatrix} 0 & M_{e\mu}^{(-)} & \sigma M_{e\mu}^{(-)} \\ M_{e\mu}^{(-)} & M_{\mu\mu}^{(-)} & 0 \\ \sigma M_{e\mu}^{(-)} & 0 & -M_{\mu\mu}^{(-)} \end{pmatrix}, \quad (\text{A2})$$

where $M_{e\mu}^{(\pm)} = (M_{e\mu} \mp \sigma M_{e\tau})/2$ and $M_{\mu\mu}^{(\pm)} = (M_{\mu\mu} \pm M_{\tau\tau})/2$. This decomposition is just an identity. However, it is so arranged that $M^{(+)}$ is invariant under the interchange $\nu_\mu \leftrightarrow -\sigma\nu_\tau$.

After the Higgses develop vacuum expectation values, the seesaw mechanism gives

$$\begin{aligned} M_{ee} &= -v^2 \left[(ch_{+e} - se^{-i\omega} h_{-e})^2 \tilde{M}_1^{-1} + (se^{i\omega} h_{+e} + ch_{-e})^2 \tilde{M}_2^{-1} \right], \\ M_{e\mu} &= -v^2 \left[(ch_{+e} - se^{-i\omega} h_{-e}) (ch_{+\mu} - se^{-i\omega} h_{-\mu}) \tilde{M}_1^{-1} + (se^{i\omega} h_{+e} + ch_{-e}) (se^{i\omega} h_{+\mu} + ch_{-\mu}) \tilde{M}_2^{-1} \right], \\ M_{e\tau} &= -v^2 \left[(ch_{+e} - se^{-i\omega} h_{-e}) (ch_{+\tau} - se^{-i\omega} h_{-\tau}) \tilde{M}_1^{-1} + (se^{i\omega} h_{+e} + ch_{-e}) (se^{i\omega} h_{+\tau} + ch_{-\tau}) \tilde{M}_2^{-1} \right], \\ M_{\mu\mu} &= -v^2 \left[(ch_{+\mu} - se^{-i\omega} h_{-\mu})^2 \tilde{M}_1^{-1} + (se^{i\omega} h_{+\mu} + ch_{-\mu})^2 \tilde{M}_2^{-1} \right], \\ M_{\tau\tau} &= -v^2 \left[(ch_{+\tau} - se^{-i\omega} h_{-\tau})^2 \tilde{M}_1^{-1} + (se^{i\omega} h_{+\tau} + ch_{-\tau})^2 \tilde{M}_2^{-1} \right], \\ M_{\mu\tau} &= -v^2 \left[(ch_{+\mu} - se^{-i\omega} h_{-\mu}) (ch_{+\tau} - se^{-i\omega} h_{-\tau}) \tilde{M}_1^{-1} + (se^{i\omega} h_{+\mu} + ch_{-\mu}) (se^{i\omega} h_{+\tau} + ch_{-\tau}) \tilde{M}_2^{-1} \right], \end{aligned} \quad (\text{A3})$$

where $\tilde{M}_\pm^{-1} = M_\pm^{-1} e^{\pm i\varphi}$, $c = \cos\theta$, $s = \sin\theta$, and $v = \langle 0|H_u|0\rangle$ for $H_u = (H_{u1}, H_{u2})^T$. It is not difficult to demonstrate that Eq.(A3) satisfies $\det(M_\nu) = 0$, which indicates the known property that the minimal seesaw model has one massless neutrino. The Yukawa couplings of $h_{\pm i}$ ($i = e, \mu, \tau$) literally represent the couplings to N_\pm . Therefore, the couplings of $h_{\pm i}$ should be expressed by the original Yukawa couplings defined in Eq.(3). For example, in the case of $N = (N_\mu, N_\tau)$, we obtain that

$$\begin{aligned} h_{+e} &= \frac{h_{\mu e} + (-\sigma) h_{\tau e}}{\sqrt{2}}, h_{+\mu} = \frac{h_{\mu\mu} + (-\sigma) h_{\tau\mu}}{\sqrt{2}}, h_{+\tau} = \frac{h_{\mu\tau} + (-\sigma) h_{\tau\tau}}{\sqrt{2}}, \\ h_{-e} &= \frac{h_{\mu e} - (-\sigma) h_{\tau e}}{\sqrt{2}}, h_{-\mu} = \frac{h_{\mu\mu} - (-\sigma) h_{\tau\mu}}{\sqrt{2}}, h_{-\tau} = \frac{h_{\mu\tau} - (-\sigma) h_{\tau\tau}}{\sqrt{2}}, \end{aligned} \quad (\text{A4})$$

where the original Yukawa couplings are h_{ij} ($i = \mu, \tau$, $j = e, \mu, \tau$).

1. N subject to the μ - τ Symmetry

In terms of Eq.(18), Eq.(A3) is expressed as,

$$\begin{aligned} M_{ee} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(-)} \right)^2 \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(-)} \right)^2 \tilde{M}_2^{-1} \right], \\ M_{e\mu}^{(+)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(-)} \right) \left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(-)} \right) \left(se^{i\omega} h_{+\mu}^{(+)} + ch_{-\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \\ M_{e\mu}^{(-)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(-)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(-)} \right) \left(se^{i\omega} h_{+\mu}^{(-)} + ch_{-\mu}^{(+)} \right) \tilde{M}_2^{-1} \right], \\ M_{\mu\mu}^{(+)} &= -v^2 \left[\left(\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(-)} \right)^2 + \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 \right) \tilde{M}_1^{-1} \right. \\ &\quad \left. + \left(\left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(-)} \right)^2 \right) \tilde{M}_2^{-1} \right], \\ M_{\mu\mu}^{(-)} &= -2v^2 \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(+)} \right) \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \end{aligned}$$

$$M_{\mu\tau} = -v^2 (-\sigma) \left[\begin{aligned} & \left(\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(-)} \right)^2 - \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 \right) \tilde{M}_1^{-1} \\ & + \left(\left(se^{i\omega} h_{+\mu}^{(+)} + ch_{-\mu}^{(-)} \right)^2 - \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(-)} \right)^2 \right) \tilde{M}_2^{-1} \end{aligned} \right], \quad (\text{A5})$$

where $c = \cos \theta$ and $s = \sin \theta$. The suffices \pm represent for (N_+, N_-) that have $N_{\pm} \rightarrow \pm N_{\pm}$ under the μ - τ symmetry transformation.

a. μ - τ Symmetry Breaking Case

The approximate μ - τ symmetry calls for

$$h_{-e}^{(-)} \approx 0, \quad h_{+\mu}^{(-)} \approx 0, \quad h_{-\mu}^{(-)} \approx 0, \quad (\text{A6})$$

as well as $\mathbf{M}_{+-} \approx 0$, which yields

$$\cos \theta \approx 1, \quad \sin \theta \approx \frac{\mathbf{M}_{R+-} r}{\mathbf{M}_{R--} - \mathbf{M}_{R++}}, \quad (\text{A7})$$

where r is defined in Eq.(14). Using these approximations, we obtain Eq.(A5) up to the first order in the parameters of Eqs.(A6) and (A7):

$$\begin{aligned} M_{ee} &\approx -v^2 h_{+e}^{(+)^2} \tilde{M}_1^{-1}, \\ M_{e\mu}^{(+)} &\approx -v^2 h_{+e}^{(+)} h_{+\mu}^{(+)} \tilde{M}_1^{-1}, \\ M_{e\mu}^{(-)} &\approx -v^2 \left[\left(h_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right) h_{+e}^{(+)} \tilde{M}_1^{-1} + \left(h_{-e}^{(-)} + se^{i\omega} h_{+e}^{(+)} \right) h_{-\mu}^{(+)} \tilde{M}_2^{-1} \right], \\ M_{\mu\mu}^{(+)} &\approx -v^2 \left(h_{+\mu}^{(+)^2} \tilde{M}_1^{-1} + h_{-\mu}^{(+)^2} \tilde{M}_2^{-1} \right), \\ M_{\mu\mu}^{(-)} &\approx -2v^2 \left[\left(h_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(+)} \right) h_{+\mu}^{(+)} \tilde{M}_1^{-1} + \left(h_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(+)} \right) h_{-\mu}^{(+)} \tilde{M}_2^{-1} \right], \\ M_{\mu\tau} &\approx -v^2 (-\sigma) \left(h_{+\mu}^{(+)^2} \tilde{M}_1^{-1} - h_{-\mu}^{(+)^2} \tilde{M}_2^{-1} \right), \end{aligned} \quad (\text{A8})$$

where $s \approx 0$.

b. μ - τ Symmetric Case

The μ - τ symmetric textures containing one massless neutrino should describe either the normal mass hierarchy or the inverted mass hierarchy. Imposing the conditions:

$$\cos \theta = 1, \quad \sin \theta = 0, \quad (\text{A9})$$

we obtain from Eq.(A5) that

$$\begin{aligned} M_{ee} (= a_0) &= -v^2 h_{+e}^{(+)^2} \tilde{M}_1^{-1}, \\ M_{e\mu} (= b_0) &= -v^2 h_{+e}^{(+)} h_{+\mu}^{(+)} \tilde{M}_1^{-1}, \\ M_{e\tau} (= c_0) &= -\sigma M_{e\mu}, \\ M_{\mu\mu} (= d_0 \equiv d_+ + d_-) &= -v^2 \left(h_{+\mu}^{(+)^2} \tilde{M}_1^{-1} + h_{-\mu}^{(+)^2} \tilde{M}_2^{-1} \right), \\ M_{\mu\tau} (= e_0 \equiv -\sigma (d_+ - d_-)) &= -v^2 (-\sigma) \left(h_{+\mu}^{(+)^2} \tilde{M}_1^{-1} - h_{-\mu}^{(+)^2} \tilde{M}_2^{-1} \right), \\ M_{\tau\tau} (= f_0) &= M_{\mu\mu}, \end{aligned} \quad (\text{A10})$$

This texture turns out to give mass terms

$$\frac{1}{2} a_0 \nu_e \nu_e + b_0 \frac{\nu_e \nu_+ + \nu_+ \nu_e}{\sqrt{2}} + d_+ \nu_+ \nu_+ + d_- \nu_- \nu_-. \quad (\text{A11})$$

This form of Eq.(A11) is also valid for the model with (N_e, N_-) . We then obtain

$$M_\nu = \begin{pmatrix} a_0 & b_0 & -\sigma b_0 \\ b_0 & d_+ + d_- & -\sigma(d_+ - d_-) \\ -\sigma b_0 & -\sigma(d_+ - d_-) & d_+ + d_- \end{pmatrix}, \quad (\text{A12})$$

where

$$\begin{aligned} a_0 &= -v^2 h_{+e}^{(+2)} \tilde{M}_1^{-1}, & b_0 &= -v^2 h_{+e}^{(+)} h_{+\mu}^{(+)} \tilde{M}_1^{-1}, \\ d_+ &= -v^2 h_{+\mu}^{(+2)} \tilde{M}_1^{-1}, & d_- &= -v^2 h_{-\mu}^{(+2)} \tilde{M}_2^{-1}. \end{aligned} \quad (\text{A13})$$

from which we observe that

$$b_0^2 = a_0 d_+ \quad (\text{A14})$$

is satisfied.

To see how the mass hierarchies are realized, it is sufficient to check the ideal case, where $m_1 = m_2 = 0$ with $m_3 \neq 0$ for the normal mass hierarchy and $m_1 = \pm m_2$ with $m_3 = 0$ for the inverted mass hierarchy. The gross structure of M_ν for the inverted mass hierarchy is described by ideal textures:

$$M_\nu^{(1)} = m_0 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -\sigma \\ 0 & -\sigma & 1 \end{pmatrix}, \quad M_\nu^{(2)} = m_0 \begin{pmatrix} -2 & b_0 & -\sigma b_0 \\ b_0 & 1 & -\sigma \\ -\sigma b_0 & -\sigma & 1 \end{pmatrix} (b_0 \neq 0), \quad (\text{A15})$$

respectively, corresponding to $m_1 = m_2$ and $m_1 = -m_2$, which can be seen from Eq.(33). Since $d_+ = m_0$ and $d_- = 0$ should be satisfied, we find that Eq.(A14) gives $0 = 2m_0^2$ for $M_\nu^{(1)}$ and $b_0^2 = -2m_0^2$ for $M_\nu^{(2)}$. Therefore, the $M_\nu^{(1)}$ case is obviously ruled out and the $M_\nu^{(2)}$ case is allowed if b_0 is nearly pure imaginary. Since no phases is present in the N mass terms, b_0 is (almost) real and the $M_\nu^{(2)}$ case is also excluded.

2. N blind to the μ - τ Symmetry

In terms of Eq.(23), Eq.(A3) is expressed as

$$\begin{aligned} M_{ee} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \\ M_{e\mu}^{(+)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(se^{i\omega} h_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) \tilde{M}_2^{-1} \right], \\ M_{e\mu}^{(-)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(se^{i\omega} h_{+\mu}^{(-)} + ch_{-\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \\ M_{\mu\mu}^{(+)} &= -v^2 \left[\left(\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 + \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right)^2 \right) \tilde{M}_1^{-1} \right. \\ &\quad \left. + \left(\left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 + \left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(-)} \right)^2 \right) \tilde{M}_2^{-1} \right], \\ M_{\mu\mu}^{(-)} &= -2v^2 \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right) \left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \\ M_{\mu\tau} &= -v^2 (-\sigma) \left[\left(\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 - \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right)^2 \right) \tilde{M}_1^{-1} \right. \\ &\quad \left. + \left(\left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 - \left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(-)} \right)^2 \right) \tilde{M}_2^{-1} \right]. \end{aligned} \quad (\text{A16})$$

The suffices \pm represent for (N_+, N_-) that have $N_\pm \rightarrow N_\pm$ under the μ - τ symmetry transformation. The heavy neutrinos (N_+, N_-) can be (N_μ, N_τ) , (N_e, N_μ) or any other combinations.

a. μ - τ Symmetry Breaking Case

The approximate μ - τ symmetry calls for

$$h_{+\mu}^{(-)} \approx 0, \quad h_{-\mu}^{(-)} \approx 0, \quad (\text{A17})$$

Using the approximation, we obtain from Eq.(A16)

$$\begin{aligned}
M_{ee} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \\
M_{e\mu}^{(+)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(se^{i\omega} h_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) \tilde{M}_2^{-1} \right], \\
M_{e\mu}^{(-)} &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(se^{i\omega} h_{+\mu}^{(-)} + ch_{-\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \\
M_{\mu\mu}^{(+)} &\approx -v^2 \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \\
M_{\mu\mu}^{(-)} &= -2v^2 \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \left(ch_{+\mu}^{(-)} - se^{-i\omega} h_{-\mu}^{(-)} \right) \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right) \left(ch_{-\mu}^{(-)} + se^{i\omega} h_{+\mu}^{(-)} \right) \tilde{M}_2^{-1} \right], \\
M_{\mu\tau} &\approx -v^2 (-\sigma) \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \tag{A18}
\end{aligned}$$

up to the first order in the parameters of Eq.(A17).

b. μ - τ Symmetric Case

We obtain from Eq.(A16) that

$$\begin{aligned}
M_{ee} (= a_0) &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \\
M_{e\mu} (= b_0) &= -v^2 \left[\left(ch_{+e}^{(+)} - se^{-i\omega} h_{-e}^{(+)} \right) \left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right) \tilde{M}_1^{-1} \right. \\
&\quad \left. + \left(se^{i\omega} h_{+e}^{(+)} + ch_{-e}^{(+)} \right) \left(se^{i\omega} h_{+\mu}^{(+)} + ch_{-\mu}^{(+)} \right) \tilde{M}_2^{-1} \right], \\
M_{e\tau} (= c_0) &= -\sigma M_{e\mu}, \\
M_{\mu\mu} (= d_0 \equiv d_+ + d_-) &= -v^2 \left[\left(ch_{+\mu}^{(+)} - se^{-i\omega} h_{-\mu}^{(+)} \right)^2 \tilde{M}_1^{-1} + \left(ch_{-\mu}^{(+)} + se^{i\omega} h_{+\mu}^{(+)} \right)^2 \tilde{M}_2^{-1} \right], \\
M_{\mu\tau} (= e_0 \equiv -\sigma (d_+ - d_-)) &= -\sigma M_{\mu\mu}, \\
M_{\tau\tau} (= f_0) &= M_{\mu\mu}, \tag{A19}
\end{aligned}$$

leading $d_- = 0$. This texture gives the following mass terms:

$$\frac{1}{2} a_0 \nu_e \nu_e + b_0 \frac{\nu_e \nu_+ + \nu_+ \nu_e}{\sqrt{2}} + d_+ \nu_+ \nu_+. \tag{A20}$$

Since $m_3 = 0$ is realized because of the relation $e_0 = -\sigma d_0$ as in Eq.(33), Eq.(A19) is only consistent with the inverted mass hierarchy. Contrary to the previous case, the normal mass hierarchy is not realized in the minimal seesaw mechanism based on N blind to the μ - τ symmetry.

APPENDIX B: FORMULA FOR MASSES, MIXINGS AND PHASES.

By adopting U_{PMNS} of Eq.(2) to diagonalize $\mathbf{M} (\equiv M_\nu^\dagger M_\nu)$, where $U_{PMNS}^\dagger \mathbf{M} U_{PMNS} = \text{diag.} (m_1^2, m_2^2, m_3^2)$ is satisfied, we can derive a set of formula to express neutrino masses and mixing angles as well as phases in terms of the flavor neutrino masses [8]. The Hermitean matrix \mathbf{M} is parameterized by $\mathbf{M} = \mathbf{M}^{(+)} + \mathbf{M}^{(-)}$ with

$$\mathbf{M}^{(+)} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & E_+ \\ -\sigma B_+^* & E_+ & D_+ \end{pmatrix}, \quad \mathbf{M}^{(-)} = \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & iE_- \\ \sigma B_-^* & -iE_- & -D_- \end{pmatrix}, \tag{B1}$$

where

$$\begin{aligned}
A &= |M_{ee}|^2 + 2 \left(|M_{e\mu}^{(+)}|^2 + |M_{e\mu}^{(-)}|^2 \right), \\
B_+ &= M_{ee}^* M_{e\mu}^{(+)} + M_{e\mu}^{(+)*} \left(M_{\mu\mu}^{(+)} - \sigma M_{\mu\tau} \right) + M_{e\mu}^{(-)*} M_{\mu\mu}^{(-)}, \\
B_- &= M_{ee}^* M_{e\mu}^{(-)} + M_{e\mu}^{(-)*} \left(M_{\mu\mu}^{(+)} + \sigma M_{\mu\tau} \right) + M_{e\mu}^{(+)*} M_{\mu\mu}^{(-)}, \\
D_+ &= |M_{e\mu}^{(+)}|^2 + |M_{e\mu}^{(-)}|^2 + |M_{\mu\mu}^{(+)}|^2 + |M_{\mu\mu}^{(-)}|^2 + |M_{\mu\tau}|^2, \\
D_- &= 2\text{Re} \left(M_{e\mu}^{(-)*} M_{e\mu}^{(+)} + M_{\mu\mu}^{(-)*} M_{\mu\mu}^{(+)} \right), \\
E_+ &= \text{Re}(E) = \sigma \left(|M_{e\mu}^{(-)}|^2 - |M_{e\mu}^{(+)}|^2 \right) + 2\text{Re} \left(M_{\mu\mu}^{(+)*} M_{\mu\tau} \right), \\
E_- &= \text{Im}(E) = 2\text{Im} \left(M_{\mu\mu}^{(-)*} M_{\mu\tau} - \sigma M_{e\mu}^{(-)*} M_{e\mu}^{(+)} \right),
\end{aligned} \tag{B2}$$

for $E = E_+ + iE_-$. Similarly, we define $B = B_+ + B_-$, $C = -\sigma(B_+ - B_-)$, $D = D_+ + D_-$, and $F = D_+ - D_-$ to describe matrix elements of \mathbf{M} . We, then, obtain that

$$\begin{aligned}
\tan 2\theta_{12} e^{i\rho} &= \frac{2X}{\Lambda_2 - \Lambda_1}, \quad \tan 2\theta_{13} e^{-i\delta} = \frac{2Y}{\Lambda_3 - A}, \\
\text{Re} \left(e^{-2i\gamma} E \right) \cos 2\theta_{23} + D_- \sin 2\theta_{23} + i\text{Im} \left(e^{-2i\gamma} E \right) &= -s_{13} e^{-i\delta} X^*,
\end{aligned} \tag{B3}$$

for three mixing angles and three phases, and

$$m_1^2 = c_{12}^2 \Lambda_1 + s_{12}^2 \Lambda_2 - 2c_{12}s_{12} |X|, \quad m_2^2 = s_{12}^2 \Lambda_1 + c_{12}^2 \Lambda_2 + 2c_{12}s_{12} |X|, \quad m_3^2 = \frac{c_{13}^2 \Lambda_3 - s_{13}^2 A}{c_{13}^2 - s_{13}^2}, \tag{B4}$$

for three masses, where

$$\begin{aligned}
X &= \frac{c_{23} e^{i\gamma} B - s_{23} e^{-i\gamma} C}{c_{13}} = \frac{e^{i\rho} |c_{23} e^{i\gamma} B - s_{23} e^{-i\gamma} C|}{c_{13}}, \\
Y &= s_{23} e^{i\gamma} B + c_{23} e^{-i\gamma} C = e^{-i\delta} |s_{23} e^{i\gamma} B + c_{23} e^{-i\gamma} C|, \\
\Lambda_1 &= \frac{c_{13}^2 A - s_{13}^2 \Lambda_3}{c_{13}^2 - s_{13}^2}, \quad \Lambda_2 = c_{23}^2 D + s_{23}^2 F - 2s_{23}c_{23} \text{Re} \left(e^{-2i\gamma} E \right), \\
\Lambda_3 &= s_{23}^2 D + c_{23}^2 F + 2s_{23}c_{23} \text{Re} \left(e^{-2i\gamma} E \right).
\end{aligned} \tag{B5}$$

In the μ - τ symmetric case, where $B_- = D_- = E_- = 0$, we obtain that $\rho = \arg(B)$, $\gamma = 0$, and $\cos 2\theta_{23} = \sin \theta_{13} = 0$. The Dirac CP violation involves the angle $\rho + \delta$.

There are useful relations:

$$\begin{aligned}
|X| &= \frac{\Delta m_\odot^2 \sin 2\theta_{12}}{2}, \quad A \approx \frac{\Sigma m_\odot^2 - \cos 2\theta_{12} \Delta m_\odot^2 + s_{13}^2 (2\Delta m_{atm}^2 - (1 - \cos 2\theta_{12}) \Delta m_\odot^2)}{2}, \\
D_+ &\approx \frac{1}{2} \left(\Delta m_{atm}^2 + \Sigma m_\odot^2 - \frac{(1 - \cos 2\theta_{12}) \Delta m_\odot^2 + s_{13}^2 (2\Delta m_{atm}^2 - (1 - \cos 2\theta_{12}) \Delta m_\odot^2)}{2} \right), \\
\sigma \text{Re} \left(e^{-2i\gamma} E \right) - 2D_- \Delta &\approx \frac{1}{2} \left(\Delta m_{atm}^2 - \frac{(1 + \cos 2\theta_{12}) \Delta m_\odot^2 + s_{13}^2 (2\Delta m_{atm}^2 - (1 - \cos 2\theta_{12}) \Delta m_\odot^2)}{2} \right), \\
\Lambda_1 &\approx \frac{\Sigma m_\odot^2 - \cos 2\theta_{12} \Delta m_\odot^2}{2}, \quad \Lambda_2 = \frac{\cos 2\theta_{12} \Delta m_\odot^2 + \Sigma m_\odot^2}{2}, \\
\Lambda_3 &\approx \frac{2\Delta m_{atm}^2 + \Sigma m_\odot^2 - \Delta m_\odot^2 - s_{13}^2 (2\Delta m_{atm}^2 - (1 - \cos 2\theta_{12}) \Delta m_\odot^2)}{2},
\end{aligned} \tag{B6}$$

up to $\mathcal{O}(\sin^2 \theta_{13})$, where $\Sigma m_\odot^2 = m_1^2 + m_2^2$. The real part of Eq.(B3)

$$\text{Re} \left(e^{-2i\gamma} E \right) \cos 2\theta_{23} + D_- \sin 2\theta_{23} = -s_{13} \cos(\rho + \delta) |X| (\equiv -z), \tag{B7}$$

determines $\cos 2\theta_{23}$, which is given by

$$\begin{aligned}\cos 2\theta_{23} &= -\frac{\kappa\sigma D_- \sqrt{\text{Re}^2(e^{-2i\gamma}E) + D_-^2 - z^2} + z\text{Re}(e^{-2i\gamma}E)}{\text{Re}^2(e^{-2i\gamma}E) + D_-^2} = \cos\left(\sigma\frac{\pi}{2} + \theta + \phi\right), \\ \cos\theta &= \sqrt{\frac{\text{Re}^2(e^{-2i\gamma}E) + D_-^2 - z^2}{\text{Re}^2(e^{-2i\gamma}E) + D_-^2}}, \quad \sin\theta = \frac{\sigma z}{\sqrt{\text{Re}^2(e^{-2i\gamma}E) + D_-^2}}, \\ \cos\phi &= \frac{\text{Re}(e^{-2i\gamma}E)}{\sqrt{\text{Re}^2(e^{-2i\gamma}E) + D_-^2}}, \quad \sin\phi = \frac{\kappa D_-}{\sqrt{\text{Re}^2(e^{-2i\gamma}E) + D_-^2}},\end{aligned}\tag{B8}$$

where κ is the sign of $\text{Re}(e^{-2i\gamma}E)$, from which we obtain that $\theta_{23} = \sigma\pi/4 + (\theta + \phi)/2$. On the other hand, the imaginary part of Eq.(B3)

$$\cos 2\gamma \text{Im}(E) - \sin 2\gamma \text{Re}(E) = s_{13} \sin(\rho + \delta) |X| (\equiv z'),\tag{B9}$$

determines γ , which is given by

$$\begin{aligned}\sin 2\gamma &= \frac{\kappa' \text{Im}(E) \sqrt{|E|^2 - z'^2} - z' \text{Re}(E)}{|E|^2} = \sin(\phi' - \theta'), \\ \cos\theta' &= \frac{\sqrt{|E|^2 - z'^2}}{|E|}, \quad \sin\theta' = \frac{z'}{|E|}, \\ \cos\phi' &= \frac{\text{Re}(E)}{|E|}, \quad \sin\phi' = \frac{\kappa' |\text{Im}(E)|}{|E|},\end{aligned}\tag{B10}$$

where κ' is the sign of $\text{Re}(E)$, from which we obtain that $\gamma = (\phi' - \theta')/2$.

The Majorana phases are calculated by the following formula derived by $U_{PMNS}^T M_\nu U_{PMNS} = \text{diag.}(m_1, m_2, m_3)$:

$$\begin{aligned}m_1 e^{-2i(\phi'_1 - \rho)} (= m_1 e^{-2i\phi_1}) &= \frac{\lambda_1 + \lambda_2}{2} - \frac{x}{\sin 2\theta_{12}}, \quad m_2 e^{-2i\phi_2} = \frac{\lambda_1 + \lambda_2}{2} + \frac{x}{\sin 2\theta_{12}}, \\ m_3 e^{-2i\phi_3} &= \frac{c_{13}^2 \lambda_3 - s_{13}^2 e^{-2i\delta} a}{c_{13}^2 - s_{13}^2},\end{aligned}\tag{B11}$$

where

$$\begin{aligned}\lambda_1 &= e^{2i\rho} \frac{c_{13}^2 a - s_{13}^2 e^{2i\delta} \lambda_3}{c_{13}^2 - s_{13}^2}, \quad \lambda_2 = c_{23}^2 e^{2i\gamma} d + s_{23}^2 e^{-2i\gamma} f - 2s_{23}c_{23}e, \\ \lambda_3 &= s_{23}^2 e^{2i\gamma} d + c_{23}^2 e^{-2i\gamma} f + 2s_{23}c_{23}e, \quad x = \frac{e^{i\rho} (c_{23}e^{i\gamma}b - s_{23}e^{-i\gamma}c)}{c_{13}}.\end{aligned}\tag{B12}$$

The CP violating Majorana phase denoted by ϕ is represented by $(\phi_2 - \phi_3)/2$ for $m_1 = 0$, leading to $K = \text{diag.}(1, e^{i\phi}, e^{-i\phi})$, and by $(\phi_1 - \phi_2)/2$ for $m_3 = 0$, leading to $K = \text{diag.}(e^{i\phi}, e^{-i\phi}, 1)$. To see the phase of M_{ee} , which affects the detection of the absolute neutrino mass $m_{\beta\beta}$ in double beta decay experiments, we have to refer to U_{PMNS} of Eq.(1) denoted by U_{PMNS}^{PDG} , which is associated with M_ν^{PDG} defined by

$$U_{PMNS}^{PDG T} M_\nu^{PDG} U_{PMNS}^{PDG} = U_{PMNS}^T M_\nu U_{PMNS},\tag{B13}$$

where M_ν^{PDG} is used in theoretical calculations compared with results of neutrino experiments. From Eq.(B13), by adjusting phases of the flavor neutrinos

$$\nu'_L = \begin{pmatrix} e^{-i\rho} & 0 & 0 \\ 0 & e^{-i\gamma} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix} \nu_L,\tag{B14}$$

for $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)^T$ used in Eq.(3), we find that

$$\begin{aligned}M_\nu^{PDG} &= \begin{pmatrix} e^{2i\rho} M_{ee} & e^{i(\rho+\gamma)} M_{e\mu} & e^{i(\rho-\gamma)} M_{e\tau} \\ e^{i(\rho+\gamma)} M_{e\mu} & e^{2i\gamma} M_{\mu\mu} & M_{\mu\tau} \\ e^{i(\rho-\gamma)} M_{e\tau} & M_{\mu\tau} & e^{-2i\gamma} M_{\tau\tau} \end{pmatrix}, \\ K^{PDG} &= \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \quad \delta_{CP} = \delta + \rho, \quad \phi_1 = \phi'_1 - \rho,\end{aligned}\tag{B15}$$

where K^{PDG} is obtained from $K = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$, as defined in Eq.(2). Therefore, it should be noted that $m_{\beta\beta}$ is equal to

$$e^{2i\rho} M_{ee}, \quad (\text{B16})$$

but not to M_{ee} .

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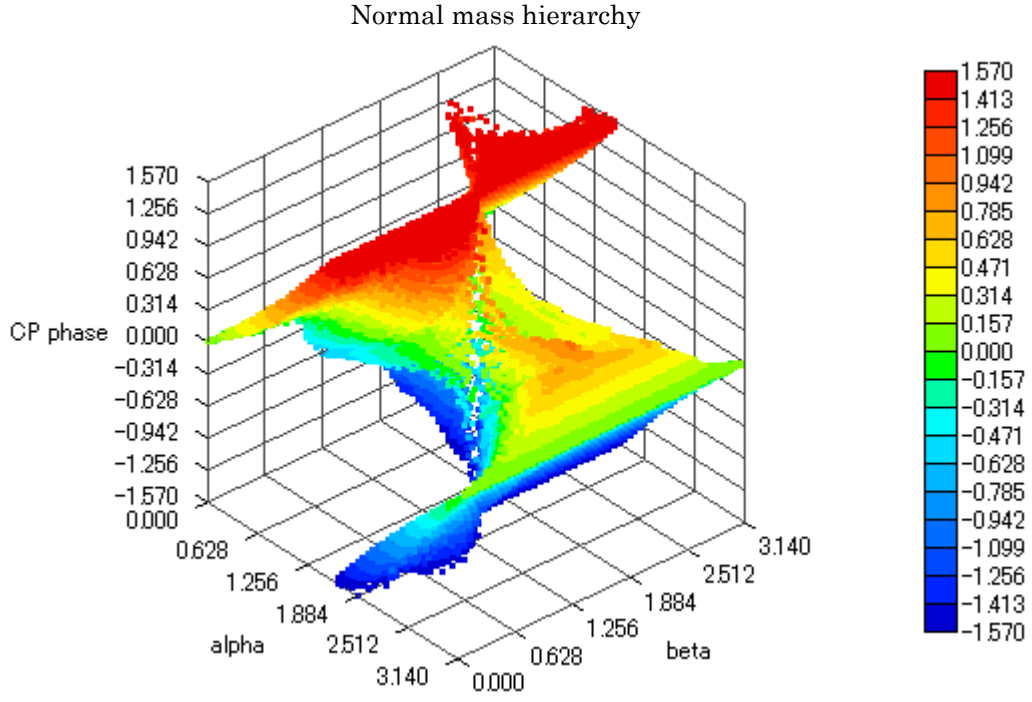


FIG. 1: The predictions of the Dirac CP phase $\delta + \rho$ as function of α and β for the normal mass hierarchy.

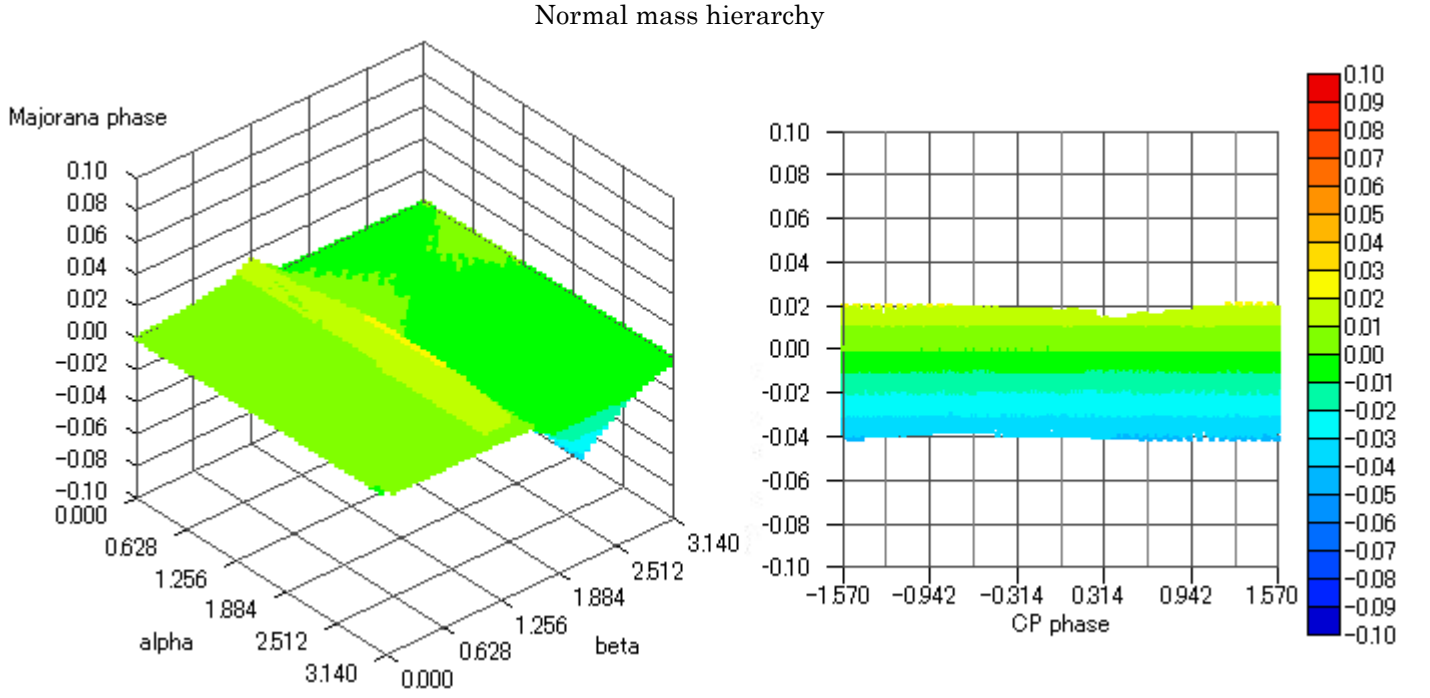


FIG. 2: The prediction of the Majorana phase as a function of the CP phase.

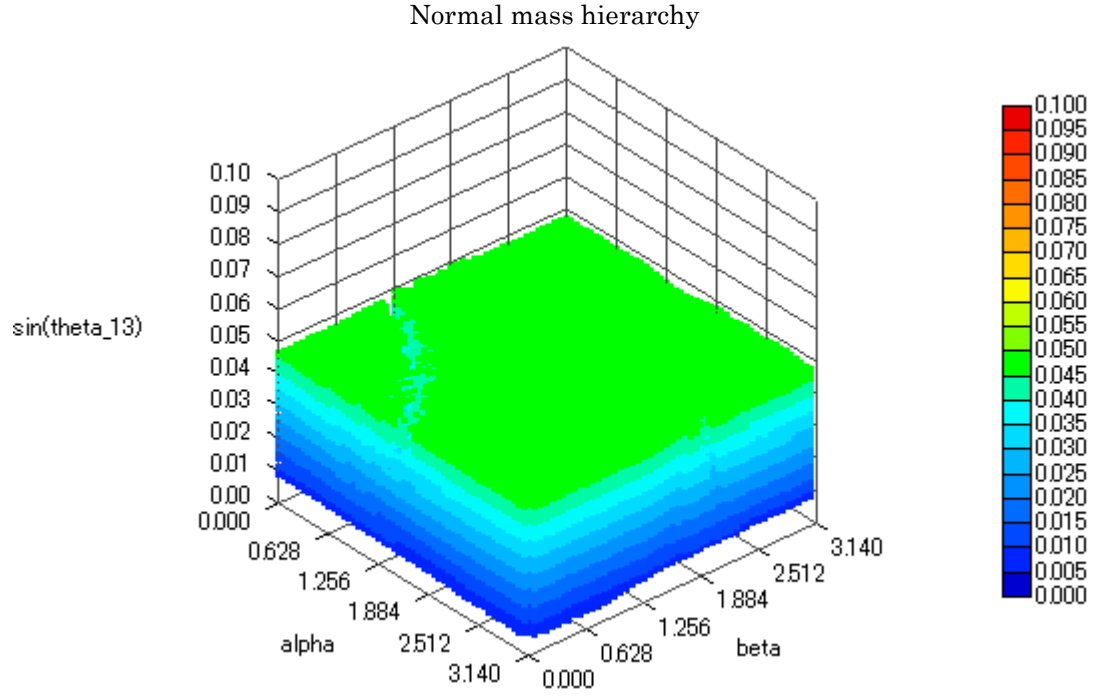


FIG. 3: The same as in FIG.1 but for $\sin \theta_{13}$.

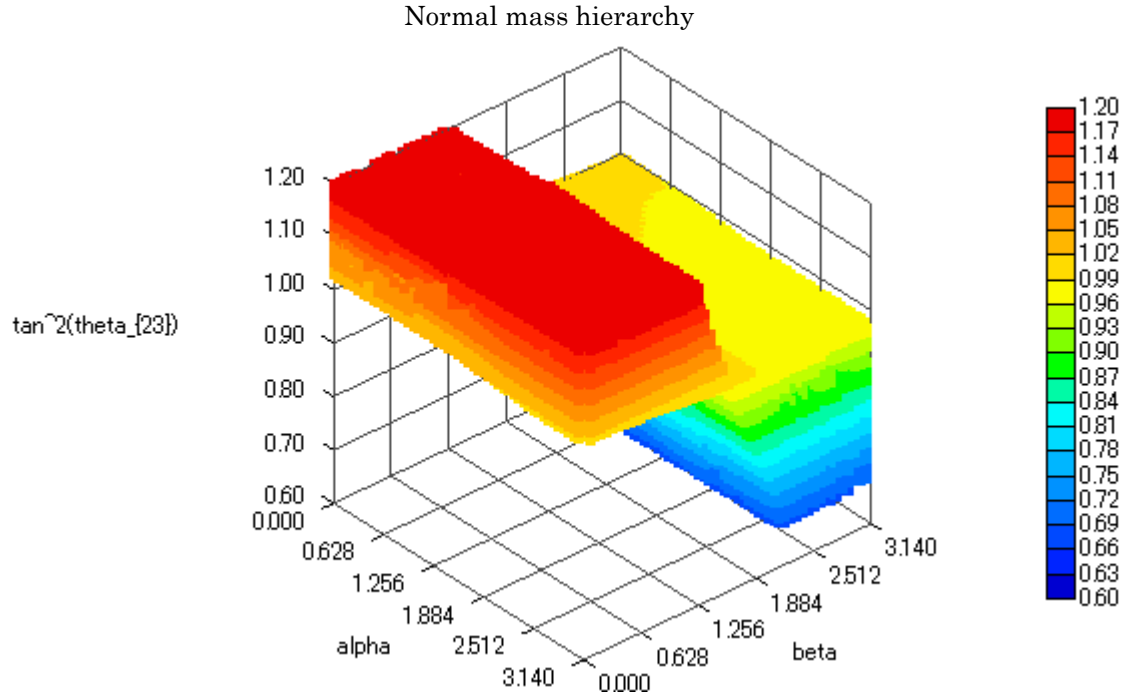


FIG. 4: The same as in FIG.1 but for $\tan^2 \theta_{23}$.

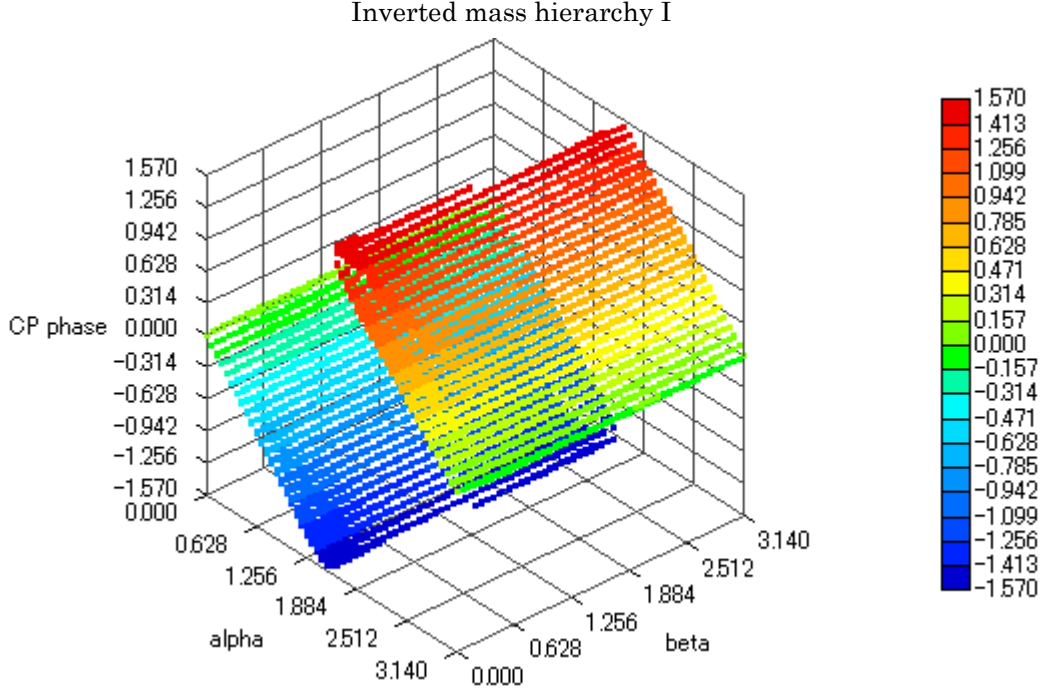


FIG. 5: The predictions of the Dirac CP phase $\delta + \rho$ as function of α and β for the inverted mass hierarchy I.

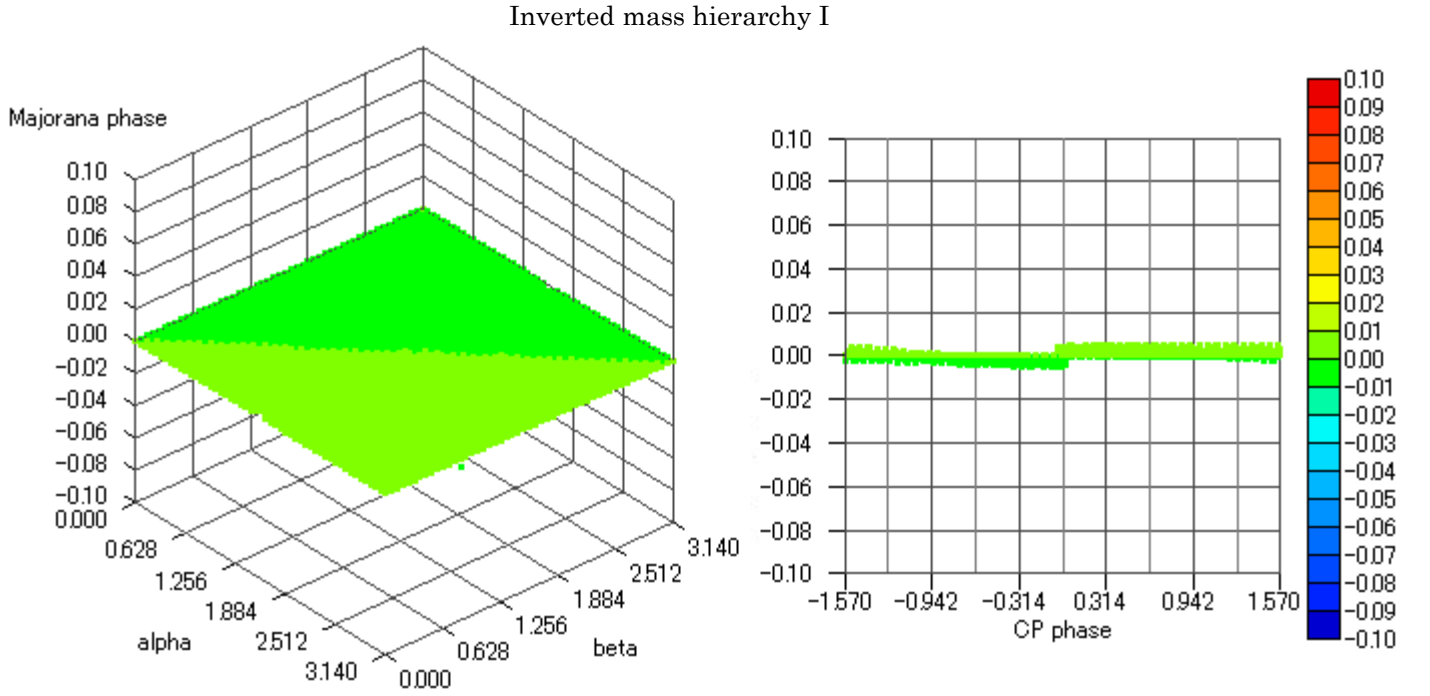


FIG. 6: The prediction of the Majorana phase as a function of the CP phase.

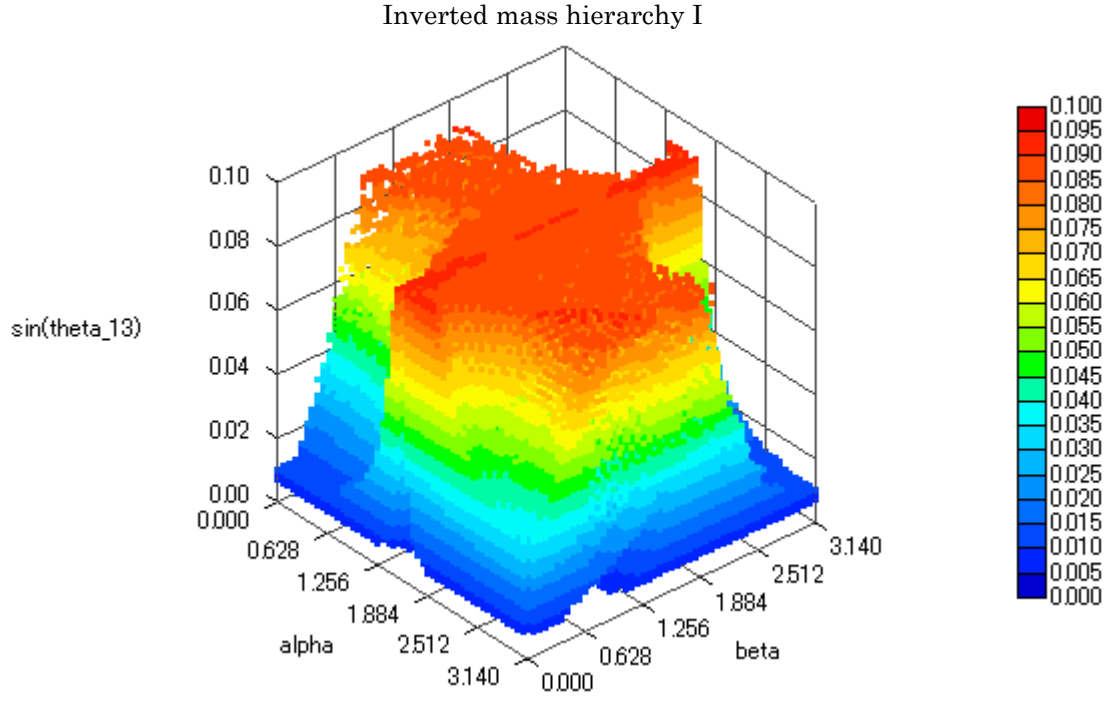


FIG. 7: The same as in FIG.5 but for $\sin \theta_{13}$.

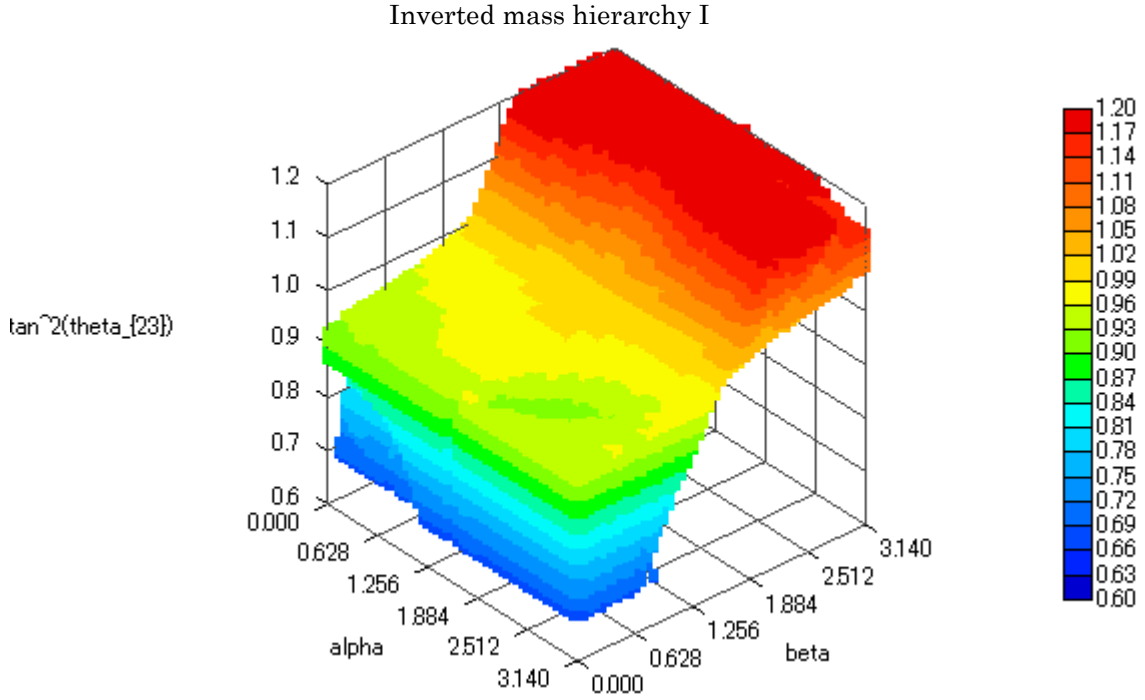


FIG. 8: The same as in FIG.5 but for $\tan^2 \theta_{23}$.

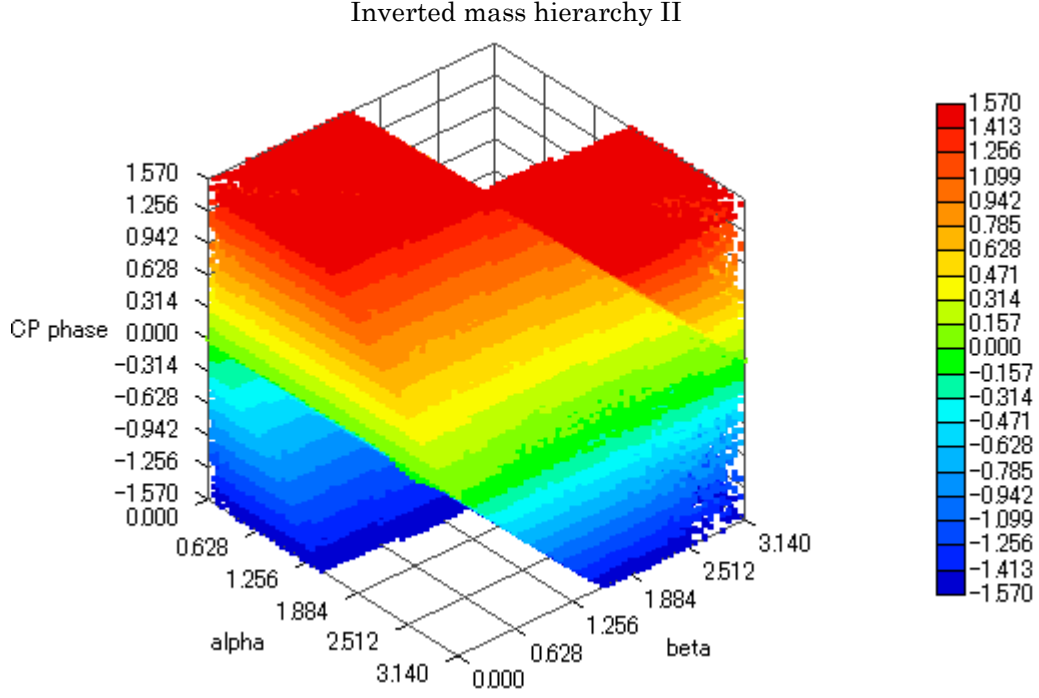


FIG. 9: The predictions of the Dirac CP phase $\delta + \rho$ as function of α and β for the inverted mass hierarchy II.

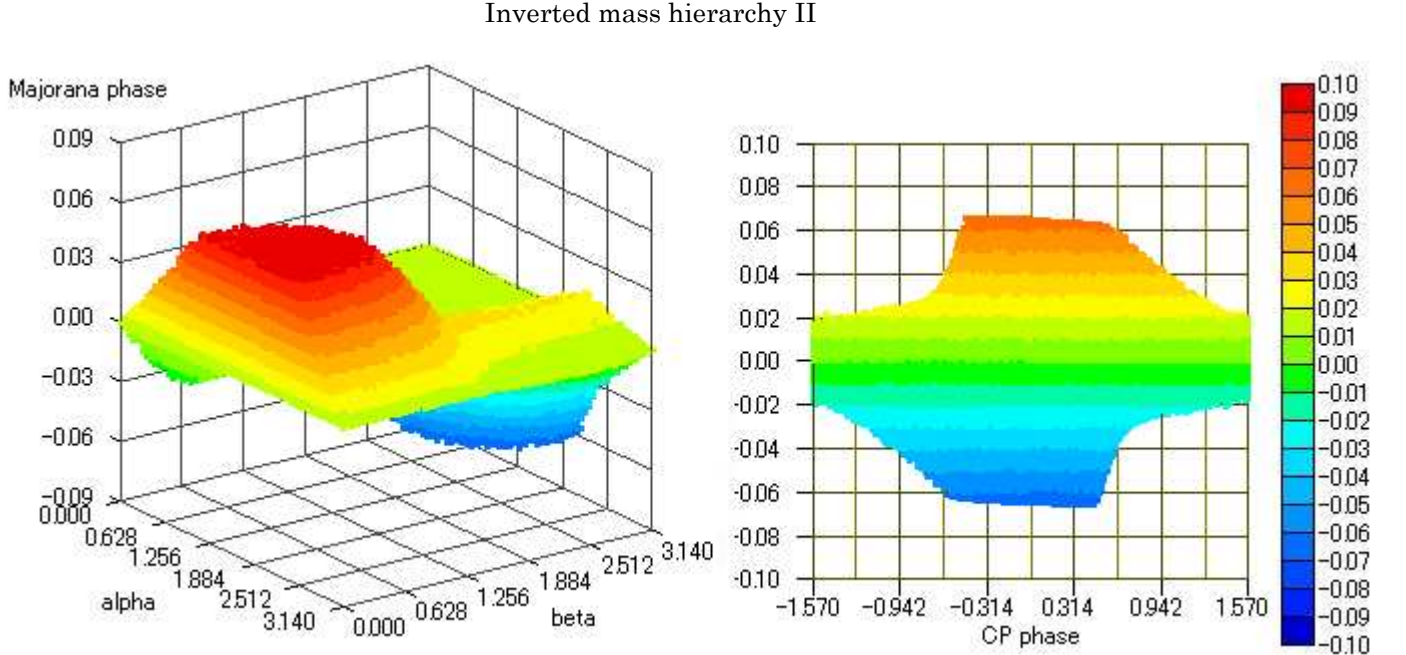
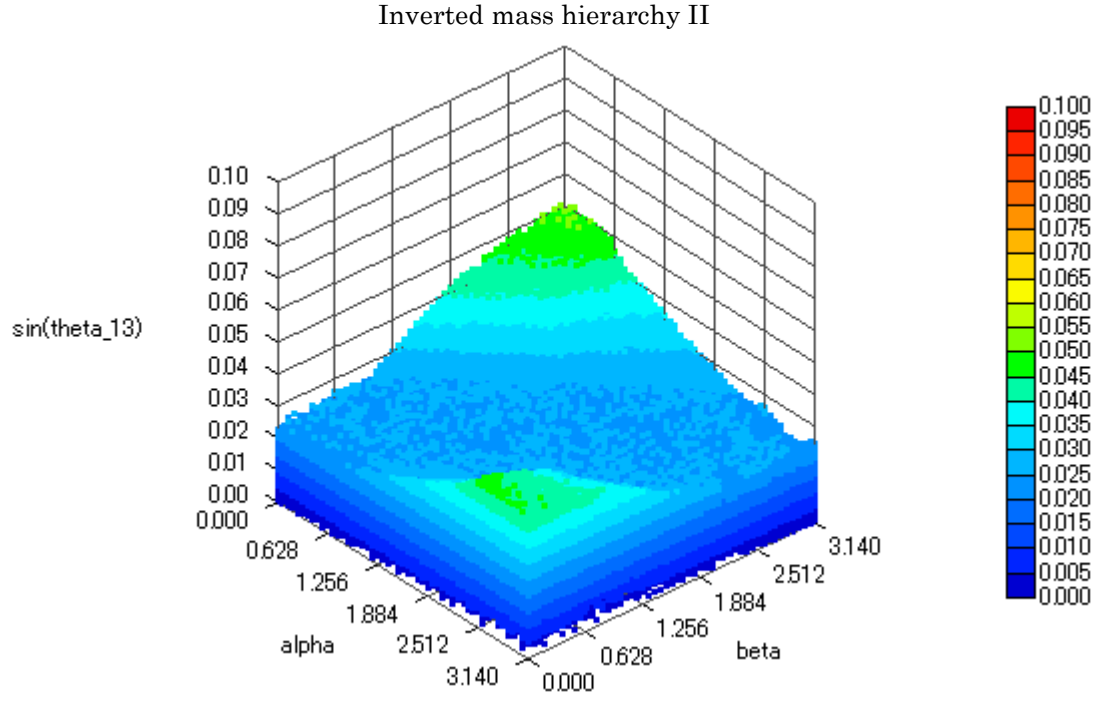
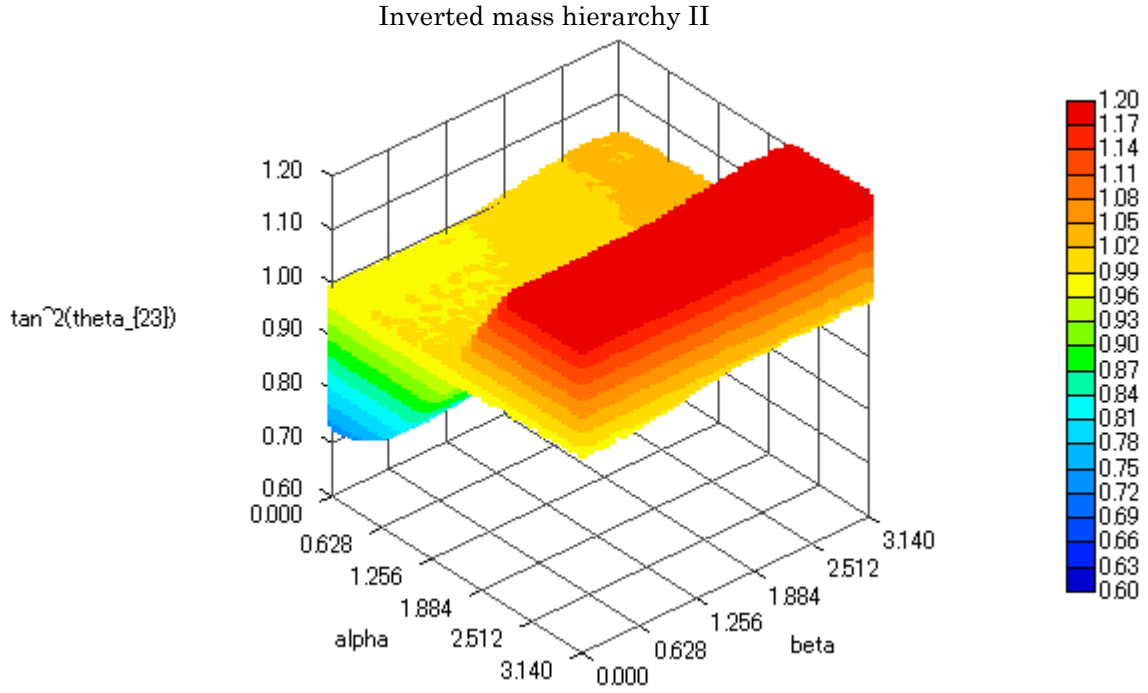
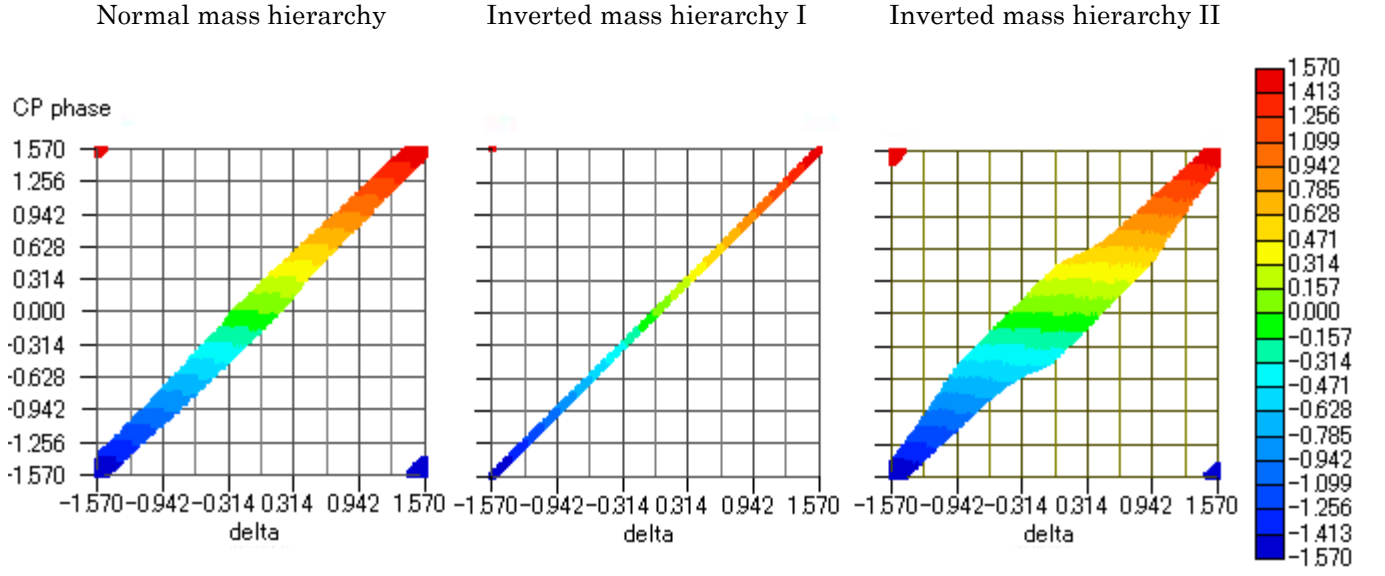
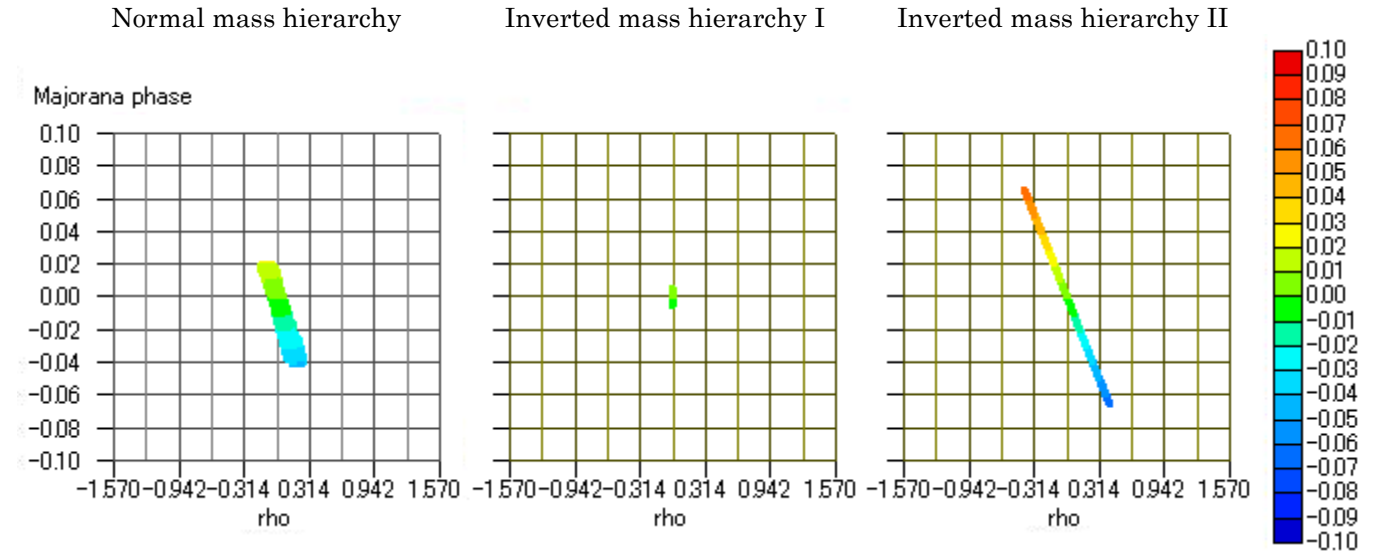


FIG. 10: The prediction of the Majorana phase as a function of the CP phase.

FIG. 11: The same as in FIG.9 but for $\sin \theta_{13}$.FIG. 12: The same as in FIG.9 but for $\tan^2 \theta_{23}$.

FIG. 13: The δ -dependence of Dirac CP phase.FIG. 14: The ρ -dependence of Majorana phase.